1. Scantron Instruction:

A. Sign your scantron sheet on the back at the bottom **in ink**.

B. Write and encode in the spaces indicated:

   1) Name(last name, first initial, middle initial)

   2) UFID Number

   3) Discussion Section Number

C. Under "special codes", code in the test ID number 2,1:

   1  ·  3  4  5  6  7  8  9  0
   o  2  3  4  5  6  7  8  9  0

D. At the top right of your scantron, for "Test Form Code", encode A.

   ·  B  C  D  E

2. This test is worth 100 points and consists of 10 + 3(bonus) multiple choice questions and 5 tear-off free response problems. The time allowed is 90 minutes.

3. When you are finished:

   1) Before turning in your test, check for transcribing errors.
      Any mistakes you leave in are there to stay.

   2) You must turn in your scantron and tear off portion to your discussion leader.

   3) Answers will be posted tomorrow on Sakai.
Be sure to bubble the answers to each question on your scantron.

Problem #1 ~ #10 are worth 5 points each.

1. Decide if the sequence $\left\{\frac{n}{\ln(n)}\right\}_{n=2}^{\infty}$ converges or diverges.

(a) converges to 0  (b) converges to 1  (c) diverges
(d) converges to 2  (e) can’t be determined

2. Which of the following series diverges?

(a) $\sum_{n=1}^{\infty} \left(\frac{3}{\pi}\right)^n$  (b) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$  (c) $\sum_{n=4}^{\infty} \frac{(-1)^n}{\ln n}$
(d) $\sum_{n=2}^{\infty} \frac{3}{n \ln n}$  (e) $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$

3. Tell which of the following series can NOT be found convergent by the ratio test, but can be found convergent by the comparison test.

(a) $\sum_{n=1}^{\infty} \frac{1}{2 + 3^n}$  (b) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 4}$
(c) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$
(d) $\sum_{n=3}^{\infty} \frac{n!}{n^n}$  (e) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n^n}$

4. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4n+3}$.

(a) $\frac{4}{7}$  (b) $-\frac{3}{7}$  (c) $-16$  (d) $\frac{1}{64}$  (e) $\frac{1}{448}$

2A
5. Let $E$ be the error made in approximating the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}$ by the first 100 terms. Then according to the Alternating Series Estimation Theorem, $|E| \leq ____.$

(a) $\frac{1}{1+(99)^2}$  
(b) $\frac{1}{1+(100)^2}$  
(c) $\frac{1}{1+(101)^2}$  
(d) $\frac{1}{1+(102)^2}$

6. Use differentiation to find a power series representation for the function $f(x) = \frac{x}{(1-x)^2}$.

(a) $\sum_{n=1}^{\infty} nx^n$  
(b) $\sum_{n=1}^{\infty} nx^{n-1}$  
(c) $\sum_{n=1}^{\infty} nx^{n+1}$

(d) $\sum_{n=0}^{\infty} nx^{n+1}$  
(e) $-\sum_{n=1}^{\infty} nx^{n-1}$

7. Find the Maclaurin series for $f(x) = \sin(2x)$.

(a) $x - \frac{2x^3}{3!} + \frac{4x^5}{5!} + \cdots$  
(b) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} + \cdots$

(c) $2(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots)$  
(d) $\frac{x}{2} - \frac{x^3}{2^3 \cdot 3!} + \frac{x^5}{2^5 \cdot 5!} + \cdots$

8. Which of the following series converges?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{(n+1)!}$  
(b) $\sum_{n=1}^{\infty} (4 + (-1)^n)$  
(c) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  
(e) $\sum_{n=2}^{\infty} \cos(n\pi)$

9. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(3-x)^n}{\sqrt{n}2^n}$.

(a) $(-1, 2)$  
(b) $(-1, 2]$  
(c) $[-1, 3)$  
(d) $[1, 5)$  
(e) $(1, 5]$
10. Investigate \( \sum_{n=1}^{\infty} \frac{\tan \left( \frac{1}{n} \right)}{n^{1/2}} \) for convergence or divergence.

(a) converges by test for divergence
(b) diverges by test for divergence
(c) converges by Ratio Test
(d) converges by limit comparison test
(e) diverges by limit comparison test

Bonus Problem #11 ~ #13 are worth 2 points each.

11. (Bonus) Determine if \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2} \) is absolutely convergent, conditionally convergent or divergent.

(a) divergent  
(b) conditionally convergent  
(c) absolutely convergent  
(d) can’t be determined

12. (Bonus) Using the Maclaurin series for \( e^x \), approximate the value of

\[ \int_0^1 e^{\sqrt{x}} \, dx \]  

\( (a) \frac{72}{15} \quad (b) \frac{31}{12} \quad (c) \frac{23}{15} \quad (d) \frac{23}{12} \quad (e) \frac{17}{3} \)

13. (Bonus) Find the coefficient of \( x^5 \) in the Maclaurin series for

\[ f(x) = \int \cos(x^2) \, dx \]

\( (a) \frac{1}{10} \quad (b) \frac{2}{15} \quad (c) \frac{1}{5} \quad (d) \frac{2}{5} \quad (e) \frac{1}{15} \)
You **MUST SHOW ALL ALGEBRAIC WORK** to receive credit.

Free response problems are worth 10 points each.

1. Using the Taylor’s formula, find the Taylor series expansion for 
   \[ f(x) = e^{-3x} \text{ at } a = 6. \]
2. (a) Using the power series \( \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \) for \( |x| < 1 \), find a power series representation for \( \ln(1+x) \).

(b) What are the radius and interval of convergence?

(c) Use the first 2 terms of the power series you obtained in part (a) to approximate the value of \( \ln(1.1) \).
3. Find the sum of the series \( \sum_{n=1}^{\infty} \left( e^{1/n} - e^{1/(n+2)} \right) \).
4. Determine if the following series converges or diverges.

Be sure to give the name(s) of the test(s) you use and to verify all necessary conditions before applying the test(s).

\[ \sum_{n=2}^{\infty} \frac{2 - \cos n}{n^2} \]
5. Determine if the following series converges or diverges.

Be sure to give the name(s) of the test(s) you use.

\[
\sum_{n=2}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n - 1)}
\]
1. Scantron Instruction:

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B. Write and encode in the spaces indicated:
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C. Under "special codes", code in the test ID number 2,2:

   1   3 4 5 6 7 8 9 0
   1   3 4 5 6 7 8 9 0

D. At the top right of your scantron, for "Test Form Code", encode B.

A   C   D   E

2. This test is worth 100 points and consists of 10 +3(bonus) multiple choice questions and 5 tear-off free response problems. The time allowed is 90 minutes.

3. When you are finished:

   1) Before turning in your test, check for transcribing errors.
      Any mistakes you leave in are there to stay.

   2) You must turn in your scantron and tear off portion to your discussion leader.

   3) Answers will be posted tomorrow on Sakai.
Be sure to bubble the answers to each question on your scantron.

Problem #1 ~ #10 are worth 5 points each.

1. Find the sum of the series \( \sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^n+3}. \)
   
   (a) \( \frac{1}{135} \)  
   (b) \( \frac{1}{448} \)  
   (c) \( \frac{4}{7} \)  
   (d) \( \frac{-3}{7} \)  
   (e) \( -16 \)

2. Which of the following series diverges?
   
   (a) \( \sum_{n=1}^{\infty} \frac{n^3}{e^n} \)  
   (b) \( \sum_{n=2}^{\infty} \frac{3}{n \ln n} \)  
   (c) \( \sum_{n=4}^{\infty} \frac{(-1)^n}{\ln n} \)  
   (d) \( \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2+1}} \)  
   (e) \( \sum_{n=1}^{\infty} \left( \frac{3}{n} \right)^n \)

3. Tell which of the following series can NOT be found convergent by the ratio test, but can be found convergent by the comparison test.
   
   (a) \( \sum_{n=1}^{\infty} \frac{n}{n^3+4} \)  
   (b) \( \sum_{n=1}^{\infty} \frac{1}{2+3^n} \)  
   (c) \( \sum_{n=3}^{\infty} \frac{n!}{n^n} \)  
   (d) \( \sum_{n=1}^{\infty} \frac{n^2}{3^n} \)  
   (e) \( \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n^n} \)

4. Decide if the sequence \( \left\{ \frac{n}{\ln(n)} \right\}_{n=2}^{\infty} \) converges or diverges.
   
   (a) converges to 0  
   (b) converges to 1  
   (c) converges to 2  
   (d) diverges  
   (e) can't be determined

2B
5. Let \( E \) be the error made in approximating the value of \( \sum_{n=1}^{\infty} \frac{(-1)^n}{1 + n^3} \) by the first 100 terms. Then according to the Alternating Series Estimation Theorem, \(|E| \leq \_\). 

\[
\begin{align*}
(a) \quad & \frac{1}{1 + (102)^3} \\
(b) \quad & \frac{1}{1 + (101)^3} \\
(c) \quad & \frac{1}{1 + (100)^3} \\
(d) \quad & \frac{1}{1 + (99)^3}
\end{align*}
\]

6. Use differentiation to find a power series representation for the function \( f(x) = \frac{x}{(1 - x)^2} \).

\[
\begin{align*}
(a) \quad & \sum_{n=0}^{\infty} n x^{n+1}\\
(b) \quad & -\sum_{n=1}^{\infty} n x^{n-1}\\
(c) \quad & \sum_{n=1}^{\infty} n x^{n}\\
(d) \quad & \sum_{n=1}^{\infty} n x^{n-1}\\
(e) \quad & \sum_{n=1}^{\infty} n x^{n+1}
\end{align*}
\]

7. Find the Maclaurin series for \( f(x) = \cos(2x) \).

\[
\begin{align*}
(a) \quad & \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \\
(b) \quad & 1 - \frac{2x^2}{2!} + \frac{4x^4}{4!} - \frac{8x^6}{6!} + \cdots \\
(c) \quad & 2(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots) \\
(d) \quad & 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \frac{64x^6}{6!} + \cdots \\
\end{align*}
\]

8. Which of the following series diverges?

\[
\begin{align*}
(a) \quad & \sum_{n=1}^{\infty} \frac{(n + 1)!}{n^n} \\
(b) \quad & \sum_{n=0}^{\infty} 3 \left(\frac{3}{4}\right)^n \\
(c) \quad & \sum_{n=1}^{\infty} (8 - (-1)^n) \\
(d) \quad & \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \\
(e) \quad & \sum_{n=2}^{\infty} \frac{1}{1 + e^n}
\end{align*}
\]

9. Find the interval of convergence of \( \sum_{n=1}^{\infty} \frac{(2 - x)^n}{\sqrt{n}3^n} \).

\[
\begin{align*}
(a) \quad & [-1, 5] \\
(b) \quad & (-1, 5] \\
(c) \quad & [-1, 3) \\
(d) \quad & (1, 5] \\
(e) \quad & [1, 5)
\end{align*}
\]

3B
10. Investigate \[ \sum_{n=2}^{\infty} \frac{\tan \left( \frac{1}{n} \right)}{n^{2/3}} \]

(a) converges by test for divergence
(b) diverges by test for divergence
(c) converges by ratio test
(d) diverges by limit comparison test
(e) converges by limit comparison test

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**Bonus Problem #11 ~ #13 are worth 2 points each.**

11. (Bonus) Determine if \[ \sum_{n=1}^{\infty} \frac{\sin(n\pi/3)}{n^3} \] is absolutely convergent, conditionally convergent or divergent.

(a) conditionally convergent (b) absolutely convergent
(c) divergent (d) can’t be determined

12. (Bonus) Using the Maclaurin series for \( e^x \), approximate the value of \[ \int_0^1 e^{x^2} \, dx \]. (use the first 3 terms).

(a) \( \frac{72}{15} \) (b) \( \frac{31}{12} \) (c) \( \frac{23}{15} \) (d) \( \frac{23}{12} \) (e) \( \frac{43}{30} \)

13. (Bonus) Find the coefficient of \( x^7 \) in the Maclaurin series for \[ f(x) = \int \sin(x^2) \, dx. \]

(a) \( -\frac{1}{10} \) (b) \( \frac{2}{15} \) (c) \( -\frac{1}{5} \) (d) \( \frac{2}{25} \) (e) \( -\frac{1}{42} \)

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4B
MAC 2312 Exam 2B  Fall 2011

Name ____________________, Section/TA ____________________

You MUST SHOW ALL ALGEBRAIC WORK to receive credit.

Free response problems are worth 10 points each.

1. Using the Taylor's formula, find the Taylor series expansion for
   \[ f(x) = e^{-2x} \text{ at } a = 5. \]
2. (a) Using the power series \( \sum_{n=0}^{\infty} x^n = \frac{1}{1 - x} \) for \(|x| < 1\), find a power series representation for \( \ln(1 + x) \).
(b) What are the radius and interval of convergence?
(c) Use the first 2 terms of the power series you obtained in part (a) to approximate the value of \( \ln(1.01) \).
3. Find the sum of the series \( \sum_{n=1}^{\infty} (e^{1/(n+2)} - e^{1/n}) \).
4. Determine if the following series converges or diverges.

Be sure to give the name(s) of the test(s) you use and to verify all necessary conditions before applying the test(s).

\[
\sum_{n=2}^{\infty} \frac{3 + \sin n}{n^3}
\]
5. Determine if the following series converges or diverges.

Be sure to give the name(s) of the test(s) you use.

\[ \sum_{n=2}^{\infty} \frac{n!}{5 \cdot 8 \cdot 11 \cdots (3n + 2)} \]