MAC2313, Calculus III
Exam 1 Review

This review is **not** designed to be comprehensive, but to be representative of the topics covered on the exam.

1. Let \( \vec{a} = \hat{i} + \hat{j} - 2\hat{k} \), \( \vec{b} = 3\hat{i} - 2\hat{j} + \hat{k} \), and \( \vec{c} = \hat{j} - 5\hat{k} \). Find

   (1) \(|\vec{a}|\)  
   (2) \(\vec{a} \cdot \vec{b}\)  
   (3) \(\vec{a} \times \vec{b}\)  
   (4) \(\vec{a} \cdot (\vec{b} \times \vec{c})\)

   (5) the angle between \( \vec{a} \) and \( \vec{b} \)  
   (6) the scalar projection of \( \vec{b} \) onto \( \vec{a} \)  
   (7) the vector projection of \( \vec{b} \) onto \( \vec{a} \)  
   (8) the area of the parallelogram determined by \( \vec{a} \) and \( \vec{b} \)  
   (9) the volume of the parallelepiped determined by \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \)

2. Three forces act on a particle as given in the diagram below. Assume the system is in equilibrium.

   ![Diagram](image)

   If \( \theta_1 = \frac{\pi}{3} \), \( \theta_2 = \frac{\pi}{6} \), \(|\vec{F}_1| = 5\)N, and \(|\vec{F}_2| = 10\)N, find (1) \(|\vec{F}_3|\) and (2) \(\theta_3\).

3. Three forces \( \vec{F}_1 = \langle 2, 1, 1 \rangle \), \( \vec{F}_2 = \langle -1, 5, 3 \rangle \) and \( \vec{F}_3 \) act on an object. Find \( \vec{F}_3 \) if the net force on the particle has magnitude 6 and is in the direction of \( \langle 1, -2, 2 \rangle \).

4. Assume that \( \vec{u} \cdot \vec{v} = -3 \) and \(|\vec{v}| = 2\). Find \( \vec{v} \cdot (2\vec{u} - 3\vec{v}) \).
5. Let \( \vec{u} = \langle 3, -1, 2 \rangle \) and \( \vec{v} = \langle -2, 1, -1 \rangle \). Express the vector \( \vec{u} \) as the sum \( \vec{u} = \vec{v}_{/\parallel} + \vec{v}_{\perp} \), where \( \vec{v}_{/\parallel} \) is parallel to \( \vec{v} \) and \( \vec{v}_{\perp} \) is perpendicular to \( \vec{v} \).

6. If \( A(1, -2, 3), \ B(-1, 4, 5), \) and \( C(0, -1, 3) \) are three points in space, find
   (1) the point closest to the \( xz \)-plane and the point closest to the plane \( x = -2 \)
   (2) an equation of the sphere with a diameter \( AB \)
   (3) a unit vector perpendicular to the plane containing \( A, \ B, \) and \( C \)
   (4) an equation of the plane containing \( A, \ B, \) and \( C \)
   (5) the area of the triangle \( ABC \)

7. (1) Determine whether \( A(1, 0, 1), \ B(2, -1, 3), \) and \( C(3, -2, 5) \) lie on the same line.
   (2) Determine whether \( P(1, 1, 1), \ Q(2, 0, 3), \ R(4, 1, 7), \) and \( S(3, -1, -2) \) lie on the same plane.

8. Do the lines \( \vec{r}_1(t) = \langle 2 + t, 1 - 2t, t + 3 \rangle \) and \( \vec{r}_2(s) = \langle 1 - s, s, 2 - s \rangle \) intersect? If so, find the point of intersection.

9. Let \( L_1 : \frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4} \) and \( L_2 : \frac{x + 1}{6} = \frac{y - 3}{-1} = \frac{z + 5}{2} \) be two lines in space.
   (1) Is \( L_1 \parallel L_2 \)? Do two lines intersect?
   (2) Find the distance from the point \((1, 1, 1)\) to \( L_1 \).

10. Let \( P_1 : \ x + y - z = 1 \) and \( P_2 : \ x - y - z = 5 \).
    (1) Do two planes intersect?
    (2) Find the angle between \( P_1 \) and \( P_2 \)
    (3) Find symmetric equations of the line of intersection of the two planes.
    (4) Find the distance from the point \((1, 1, -1)\) to \( P_1 \).

11. Discuss traces of the surface \( x^2 - y^2 + 4z^2 + 2y = 1 \) and identify the surface.
12. Find an equation of the surface consisting of all points $P(x, y, z)$ that are equidistant from $P$ to the $z$-axis and from $P$ to the plane $x = -1$. Identify the surface.

13. Consider the curve $\vec{r}(t) = \cos t \hat{i} + t \hat{j} - \sin t \hat{k}$. Find

(1) the unit tangent vector $\hat{T}(t)$ and the unit normal vector $\hat{N}(t)$
(2) the tangent line to the curve at $(1, 0, 0)$
(3) the arc length from $(1, 0, 0)$ to $(1, 2\pi, 0)$
(4) $\frac{ds}{dt}$
(5) the curvature of the curve at the point $(1, 0, 0)$

14. Find the curvature of the function $y = x^4$ at the point $(1, 1)$.

15. Let $\vec{r}(t) = \langle t^2, 2t, \ln(t) \rangle$ be a vector function describes the path of a particle with respect to $t$. Find the tangential and normal components of acceleration at $t = 1/2$.

16. For the curve given by $\vec{r}(t) = \langle \sin^3 t, \cos^3 t, \sin^2 t \rangle$, $0 < t < \pi/2$, find

(1) the unit tangent vector
(2) the unit normal vector
(3) the unit binormal vector
(4) the curvature

17. Let $\vec{r}(t) = \langle t \ln(t), \sin(\pi t), \sqrt{5 - t} \rangle$ be a vector function.

(1) Find the domain of $\vec{r}(t)$.
(2) Find $\lim_{t \to 0^+} \vec{r}(t)$.
(3) Find $\int \vec{r}(t) \, dt$.
(4) Let the curve $C$ be parametrized by $\vec{r}(t)$. Find $a$ and $b$ if the vector $\langle a, b, 1 \rangle$ is parallel to the tangent vector of the curve $C$ at the point $(0, 0, 2)$. 