This review includes typical exam problems. It is not designed to be comprehensive, but to be representative of the topics covered on test 2. You may also pick up the Fall 2016 exam at Broward Teaching center and work the Fall 2013 exam in the binder from Target Copy. However, those exams cover L11 - 19 only, so you should also work problems 1 - 4, 5 – 7 and 14 MC on Exam 3 in the binder. Remember you have 90 minutes for your exam, so you should pay attention to time as you practice for the test.

1. Use the definition of derivative to evaluate $f'(x)$ if $f(x) = \sqrt{2x - 1}$. Check your answer using a derivative rule.

2. (a) Use the **definition of derivative** to find $f'(x)$ if $f(x) = \frac{x}{2x - 1}$. Check your answer using the Quotient Rule.

   (b) Find each interval over which $f(x)$ is differentiable.

   (c) Write the equation of the tangent line to $f(x) = \frac{x}{2x - 1}$ at $x = -1$.

3. Indicate whether each of the following statements is true or false.

   (a) If $f$ is continuous at $x = a$, then $f$ is differentiable at $x = a$.

   (b) If $f$ is not continuous at $x = a$, then $f$ is not differentiable at $x = a$.

   (c) If $f$ has a vertical tangent line at $x = a$, then the graph of $f'(x)$ has a vertical asymptote at $x = a$.

4. Let $f(x) = \begin{cases} 2 - x|x| & x < 0 \\ 3x + 2 & x \geq 0 \end{cases}$.

   (a) Use the limit definition of continuity to show that $f(x)$ is continuous at $x = 0$.

   (b) Find $f'(0)$ if possible using the **limit definition** of derivative at a point

   $$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}.$$  

5. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height $h$ in feet above the ground after $t$ seconds is given by $h(t) = 200 + 64t - 16t^2$. Find the following:

   (a) The average velocity of the object from time $t = 0$ until it reaches its maximum height (hint: consider the graph of the function)

   (b) The instantaneous velocity of the object at time $t = 1$ second using the limit definition.
6. Find each value at which \( f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x \) is parallel to the line \( 2y - 8x + 9 = 0 \).

7. Find the value of \( a \) so that the tangent line to \( y = x^2 - 2\sqrt{x} + 1 \) is perpendicular to the line \( ay + 2x = 2 \) when \( x = 4 \).

8. If \( f(x) = (x^3 - 2x)(2\sqrt{x} + 1) \), find \( f'(x) \) two ways: rewriting \( f(x) \) and differentiating, and using the Product Rule.

9. Find each value of \( x \) at which \( f(x) = (1 - x)^5(5x + 2)^4 \) has a horizontal tangent line.

10. Let \( f(x) = \frac{(\sqrt{x} - 1)^2}{x} \). Find \( f'(x) \) and write as a single fraction. Write the equation of the tangent line to \( f(x) \) at \( x = 4 \).

11. Write the equation of the tangent line to \( f(x) = \left( x - \frac{6}{x} \right)^3 \) at \( x = 3 \).

12. Find each value of \( x \) at which the function \( f(x) = \frac{\sqrt{6x + 1}}{x} \) has
   (a) horizontal and (b) vertical tangent lines.
   Write the equation of each of those lines.

13. Suppose that \( f(4) = -1 \), \( g(4) = 2 \), \( f(-4) = 1 \), \( g(-4) = 3 \), \( f'(4) = -2 \), \( g'(4) = 12 \), \( f'(-4) = 6 \), and \( g'(-1) = -2 \).
   Find: (a) \( h'(4) \) if \( h(x) = g(f(x)) \) and (b) \( H'(4) \) if \( H(x) = \sqrt{x f(x) + \frac{x^2}{2}} \).

14. Find \( \frac{dy}{dx} \) if \( x^3 - 3x^2y = 1 - y^3 \).

15. The demand function for a certain product is given by \( p(x) = -0.02x + 400 \), \( 0 \leq x \leq 20,000 \), where \( p \) is the unit price when \( x \) items are sold.
   (a) Find the revenue function \( R(x) \). Find the marginal revenue when \( x = 300 \).
      What happens when \( x = 15,000 \)?
   (b) Suppose the cost function for the product is \( C(x) = 100x + 300,000 \). Find the profit function. What is the marginal profit when \( x = 2000 \)?
   (c) Find the actual profit from the sale of the 2001st item. Compare to your answer in (b).
   (d) Find each interval on which the profit function is increasing and decreasing. Remember that \( 0 \leq x \leq 20,000 \). Hint: what is the sign of the derivative when profit is increasing and decreasing?
   (e) How many items should be sold to maximize profit? At what price?
      (Hint: consider part (d))
16. The number of gallons of water in a tank $t$ minutes after the tank has started to drain is given by $Q(t) = 200(30 - t)^2$.

(a) How much water is in the tank 10 minutes after it has started to drain?

(b) How fast is the water running out of the tank at $t = 10$ minutes?

(c) Use parts (a) and (b) to approximate the amount of water in the tank one minute later ($t = 11$). Compare to the actual amount of water in the tank at that time.

17. A pollutant from a factory is carried away by wind currents. Its concentration in the air $x$ miles from the factory is given by $P(x) = \frac{1.6}{3x - 2}$ where $P(x)$ is measured in parts per million.

(a) What is the average rate at which the concentration is changing when the pollutant has moved from 1 to 4 miles away from the factory? Include units in your answer.

(b) At what rate is the concentration changing when the pollutant is two miles from the factory? Include units.

(c) Now suppose that the distance of the pollutant from the factory is given by $x(t) = t^2 + t$ where $t$ is in hours. Find the rate at which the concentration of the pollutant is changing with respect to time after 2 hours.

18. The cost function for a product is $C(x) = 1.25x^2 + 25x + 8000$.

(a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.

(b) Average cost is defined to be $\frac{C(x)}{x}$. Find and interpret marginal average cost at a production level of 50 units.

19. The demand function for a product is $p(x) = 45 - \frac{\sqrt{x}}{2}$ where $p$ is the price at which $x$ items will sell.

(a) Find the marginal revenue when 1600 items are produced.

(b) Now suppose that when 1600 items are produced in a week, revenue increases by $450 per week. At what rate is the production level $x$ changing with respect to time at that production level?

20. Sand is falling off of a conveyor belt in a conical shape at the rate of 30 ft$^3$/min. in such a way that the radius of the cone remains approximately three-fourths of its height. At what rate is the height of the pile changing when it is 8 feet high? Recall that the volume of a cone is given by $V(r) = \frac{1}{3}\pi r^2 h$. 
21. An observer is standing 50 feet from a helicopter launch pad. A helicopter lifts off vertically and rises at a rate of 40 ft/sec. At what rate is the distance between the observer and the helicopter changing at the instant when the helicopter is 120 feet high?

22. Sketch a possible graph of the derivative of the function $y = f(x)$ shown below.

![Graph of $y = f(x)$](image)

23. Find the derivative of $f(x) = 3^{2x-1}$.

24. Find the slope of the tangent line to the curve given by
$$\sqrt{3x - y} - e^{x+y} = 1 + \ln x$$ at $(1, -1)$. 