1. Use the following graph of a function $f(x)$ to evaluate the limits and function value if possible. If the limit does not exist, write “dne”.

![Graph of f(x)](image)

a) $\lim_{x \to 0^-} f(x)$  
b) $\lim_{x \to 0^+} f(x)$  
c) $\lim_{x \to 0} f(x)$  
d) $\lim_{x \to 1^+} f(x)$  
e) $\lim_{x \to 1^-} f(x)$  
f) $\lim_{x \to 1} f(x)$  
g) $\lim_{x \to 3} f(x)$  
h) $f(3)$  
i) $\lim_{x \to 0^-} f(x)$

2. Use the properties of limits to evaluate $\lim_{x \to a} (fg)(x)$ if $\lim_{x \to a} f(x) = -\frac{1}{3}$ and $\lim_{x \to a} g(x) = 9$.

3. Evaluate (a) $\lim_{x \to -1} \frac{x + \sqrt{x + 2}}{x + 1}$ and (b) $\lim_{x \to -2} \frac{2}{x - 1}$.

4. If $f(x) = \begin{cases} \frac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases}$

find $p = \lim_{x \to -4} f(x)$ and $q = \lim_{x \to -1^-} f(x)$.

5. Sketch the graph of $f(x) = \frac{|6 - 2x|}{x - 3}$. Hint: rewrite as a piecewise function without absolute value bars. Use the graph to find: (a) $\lim_{x \to 3^-} f(x)$, (b) $\lim_{x \to 3^+} f(x)$, and (c) $\lim_{x \to 3} f(x)$.

Now find those limits algebraically without using the graph.

6. If $f(x) = \frac{x^3 + 3x^2 + 2x}{x - x^3}$, find a) $\lim_{x \to 0^+} f(x)$ b) $\lim_{x \to -1^+} f(x)$ c) $\lim_{x \to -1^-} f(x)$ and d) $\lim_{x \to -\infty} f(x)$. Find each vertical and horizontal asymptote of $f(x)$. 
7. If \( f(x) = \frac{2}{e^{-x} - 3} \), find if possible:

1) \( \lim_{x \to -\infty} f(x) \)  
2) \( \lim_{x \to +\infty} f(x) \)  
3) Each asymptote of the graph of \( f(x) \).

8. The Intermediate Value Theorem guarantees that the function

\( f(x) = x^3 - \frac{1}{x} - 5x + 3 \) has a zero on which of the following intervals?

- a) \([-1, 1]\)
- b) \([1, 3]\)
- c) \([3, 5]\)
- d) \([-3, -2]\)

9. Consider a function \( f(x) \) which has the following graph.

![Graph of a function with various points and asymptotes.

(a) On which interval(s) is \( f(x) \) continuous?
(b) \( f(x) \) has a jump discontinuity at \( x = \) ________.
(c) \( f(x) \) has an infinite discontinuity at \( x = \) ________.
(d) \( f(x) \) has a removable discontinuity at \( x = \) ________.
(e) How would you define or redefine \( f(x) \) at the point(s) in part (d) in order to make \( f(x) \) continuous?
(f) Find each value at which \( f(x) \) is continuous but not differentiable.
(g) Find \( f'(-1) \).
(h) Which is larger, \( f'(-5) \) or \( f'(-3) \)?

10. Use the definition of derivative to evaluate \( f'(x) \) if \( f(x) = \sqrt{2x - 1} \). Check your answer using a derivative rule.

11. (a) Use the \textbf{definition of derivative} to find \( f'(x) \) if \( f(x) = \frac{x}{2x - 1} \). Check your answer using the Quotient Rule.

(b) Find each interval over which \( f(x) \) is differentiable.

(c) Write the equation of the tangent line to \( f(x) = \frac{x}{2x - 1} \) at \( x = -1 \).
12. Indicate whether each of the following statements is true or false.
   (a) If \( f \) is continuous at \( x = a \), then \( f \) is differentiable at \( x = a \).
   (b) If \( f \) is not continuous at \( x = a \), then \( f \) is not differentiable at \( x = a \).
   (c) If \( f \) has a vertical tangent line at \( x = a \), then the graph of \( f'(x) \) has a vertical asymptote at \( x = a \).

13. Let \( f(x) = \begin{cases} 2 - x|x| & x < 0 \\ 3x + 2 & x \geq 0 \end{cases} \).
   (a) Use the limit definition of continuity to show that \( f(x) \) is continuous at \( x = 0 \).
   (b) Find \( f'(0) \) if possible using the limit definition of derivative at a point
   \[ f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}. \]

14. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height \( h \) in feet above the ground after \( t \) seconds is given by
   \( h(t) = 200 + 64t - 16t^2 \). Find the following:
   (a) The average velocity of the object from time \( t = 0 \) until it reaches its maximum height (hint: consider the graph of the function)
   (b) The instantaneous velocity of the object at time \( t = 1 \) second using the limit definition.

15. Find each value at which \( f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x \) is parallel to the line \( 2y - 8x + 9 = 0 \).

16. Find the value of \( a \) so that the tangent line to \( y = x^2 - 2\sqrt{x} + 1 \) is perpendicular to the line \( ay + 2x = 2 \) when \( x = 4 \).

17. If \( f(x) = (x^3 - 2x)(2\sqrt{x} + 1) \), find \( f'(x) \) two ways: rewriting \( f(x) \) and differentiating, and using the Product Rule.

18. Find each value of \( x \) at which \( f(x) = (1 - x)^5(5x + 2)^4 \) has a horizontal tangent line.

19. Let \( f(x) = \frac{(\sqrt{x} - 1)^2}{x} \). Find \( f'(x) \) and write as a single fraction. Write the equation of the tangent line to \( f(x) \) at \( x = 4 \).

20. Write the equation of the tangent line to \( f(x) = \left(x - \frac{6}{x}\right)^3 \) at \( x = 3 \).
21. Find each value of $x$ at which the function $f(x) = \frac{\sqrt{6x+1}}{x}$ has
(a) horizontal and (b) vertical tangent lines.
Write the equation of each of those lines.

22. Suppose that $f(4) = -1$, $g(4) = 2$, $f(-4) = 1$, $g(-4) = 3$, $f'(4) = -2$,
$g'(4) = 12$, $f'(-4) = 6$, and $g'(-1) = -2$.
Find: (a) $h'(4)$ if $h(x) = g(f(x))$ and (b) $H'(4)$ if $H(x) = \sqrt{x}f(x) + \frac{x^2}{2}$.

23. Sketch a possible graph of the derivative of the function $y = f(x)$ shown below.