1. Find and sketch the domain of the function.

(1) \( f(x, y) = \ln(x + y + 1) \)
(2) \( f(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2} \)

2. Show that the limit does not exist.

(1) \( \lim_{(x,y) \to (1,1)} \frac{xy^2 - 1}{y - 1} \)
(2) \( \lim_{(x,y) \to (0,0)} \frac{x^3 + y^3}{xy^2} \)

3. Evaluate the following limits.

(1) \( \lim_{(x,y) \to (1,1)} \frac{x^3y^3 - 1}{xy - 1} \)
(2) \( \lim_{(x,y) \to (2,2)} \frac{x + y - 4}{\sqrt{x + y} - 2} \)
(3) \( \lim_{(x,y) \to (0,0)} \frac{e^y \sin(2x)}{x} \)
(4) \( \lim_{(x,y) \to (0,0)} \frac{(x^2 + y^2) \ln(x^2 + y^2)}{x^2} \)

4. The contour map of a function \( f \) is shown.

(1) Is \( f_x(3, 2) \) positive or negative?
(2) Which is greater, \( f_y(2, 1) \) or \( f_y(2, 2) \)?
5. Consider the function \( f(x, y) = \begin{cases} \frac{\sin(xy)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} \).

(1) Is \( f \) continuous at \((0, 0)\)?

(2) Is \( f \) differentiable at \((0, 0)\)?

6. Consider the function \( f(x, y) = \begin{cases} \frac{x^2y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases} \).

(1) Is \( f \) continuous at \((0, 0)\)?

(2) Can you redefine the function so that \( f \) is continuous at \((0, 0)\)?

7. Find all the first and second order partial derivatives of \( f(x, y) = x^y \).

8. Find the linear approximation of the function \( f(x, y, z) = x^3\sqrt{y^2 + z^2} \) at the point \((2, 3, 4)\) and use it to estimate the number \((1.98)^3\sqrt{(3.02)^2 + (4.01)^2}\).

9. Use differentials to estimate the amount of metal in a closed cylindrical can that is 30 cm high and 5 cm in radius if the metal in the top and the bottom is 0.3 cm thick and the metal in the sides is 0.05 cm thick.

10. Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) at \((0, 1, 2)\) if \( x - yz + \cos(xyz) = 2 \).

11. Find an equation of the tangent plane to the surface \( z = x\sin(x + y) \) at the point \((-1, 1, 0)\).

12. Let \( z = \sqrt{x^2 + y^2} \). Show that \( \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} = \left( \frac{\partial^2 z}{\partial x \partial y} \right)^2 \).

13. Find \( \frac{\partial w}{\partial r} \) and \( \frac{\partial w}{\partial \theta} \) when \( r = 2 \) and \( \theta = \pi/2 \) if \( w = xy + yz + zx \), and \( x = r\cos \theta, \ y = r\sin \theta, \ z = r\theta \).
14. Find the directional derivative of \( f(x, y) = x^2e^{-y} \) at the point \((-2, 0)\) in the direction toward the point \((2, -3)\).

15. Let \( f(x, y) = \ln(1 + xy) \).

(1) Find the unit vectors that give the direction of steepest ascent and steepest descent at \((1, 2)\).
(2) Find a unit vector that points in a direction of no change at \((1, 2)\).

16. Find equations of (1) the tangent plane and (2) the normal line to the surface \( xy + yz + zx = 5 \) at the point \((1, 2, 1)\).

17. Where does the normal line to the paraboloid \( z = x^2 + y^2 \) at the point \((1, 1, 2)\) intersect the paraboloid a second time?

18. The plane \( y + z = 3 \) intersects the cylinder \( x^2 + y^2 = 5 \) in an ellipse. Find parametric equations for the tangent line to this ellipse at the point \((1, 2, 1)\).

19. Find the points on the surface \( 2x^3 + y - z^2 = 5 \) at which the tangent plane is parallel to the plane \( 24x + y - 6z = 3 \).

20. Let \( f(x, y) = 3x^2 - 3xy^2 + y^3 + 3y^2 \). Find the critical points of \( f \) and classify each critical point.

21. Find the local maximum and minimum values and saddle point(s) of the function \( f(x, y) = (x^2 + y^2)e^{-x} \).

22. Find the absolute maximum and minimum values of

(1) \( f(x, y) = x^2 + y^2 - 2x \) on the closed triangular region with vertices \((2, 0)\), \((0, 2)\), and \((0, -2)\)

(2) \( f(x, y) = (x^2 + 2y^2)e^{-x^2-y^2} \) on the disk \( \{(x, y) \mid x^2 + y^2 \leq 4\} \)

(3) \( f(x, y) = e^{-xy} \) on \( \{(x, y) \mid x^2 + 4y^2 \leq 1\} \)
23. Find the maximum and minimum values of
(1) \( f(x, y, z) = x + y + z \) subject to \( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 \)
(2) \( f(x, y, z) = x^2 + y^2 + z^2 \) subject to \( x - y = 1 \) and \( y^2 - z^2 = 1 \)

24. Find the point(s) on the surface \( x^2 - yz = 1 \) that are closest to the origin.

25. Find the point on the ellipse \( x^2 + 6y^2 + 3xy = 40 \) with the largest \( x \) coordinate.

26. True or False:
(1) There exists a function \( f \) with continuous second partial derivatives such that \( f_x = x + y^2 \) and \( f_y = x - y^2 \).
(2) If \( f_x(a, b) \) and \( f_y(a, b) \) both exist, then \( f \) is differentiable at \( (a, b) \).
(3) If \( f(x, y) \) is differentiable, then the rate of change of \( f \) at the point \( (a, b) \) in the direction of \( \vec{w} \) is \( \nabla f(a, b) \cdot \vec{w} \).
(4) If \( f_x(a, b) = 0 \) and \( f_y(a, b) = 0 \), then \( f \) must have a local maximum or minimum at \( (a, b) \).
(5) If \( f(x, y) \) is differentiable and \( f \) has a local minimum at \( (a, b) \), then \( D_uf(a, b) = 0 \) for any unit vector \( \vec{u} \).