A Completing the Square: Here is a little more practice in completing the square. Since we can’t really do most of the really interesting complex numbers stuff without it, you should really make sure to be good with this before you move on.

1. $9x^4 - 42x^2 - 90$
2. $100z^6 + 60z^3$
3. $x^2 + 7x + 5$
4. $x^6 - 26x^3 + 105$
5. $64r^2 + 4r - \frac{1}{2}$
6. $9\lambda^8 - 10\lambda^3 + \frac{16}{9}$
7. $7x^8 + 42x^4 + 7$
8. $5x^6 - 20x^3 + 45$
9*. Solve for $x$ in the following equation: $64x^2 + 24x + 7 = 10$
10*. Solve for $x$ in the following equation: $4x - 6 = -\frac{10}{x}$

B Complex Arithmetic: This section is largely just some practice with the mechanics of multiplication with complex numbers. Hopefully some practice with this will make life a little less awful when you are actually doing arithmetic with complex numbers.

1. Simplify $i^2$
2. Simplify $i^5$
3. Simplify $i^7$
4. Simplify $i^{16}$
5. Multiply $(5 + 6i)(7 - 2i)$
6. Multiply $(\frac{1}{2} + 3i)(4 - \frac{1}{2}i)$
7. Multiply $2 + 2i$ by its complex conjugate
8. Multiply $1 - \frac{3}{2}i$ by its complex conjugate

C Complex Polynomials: Here we are going to investigate some basic properties of polynomials with complex roots. This is going to get a little crazy, but I hope that it stays only a mild amount of crazy as opposed to full on bonkers. Remember that our approach to complex numbers has largely been as zeros of polynomials that were
irreducible in the real numbers. As a result most of our solutions are going to be taking common forms or transforms and just going a little further with them.

1 Find all the real and complex roots of \( p_4(x) = x^4 - 1 \)
2 Find all the real and complex roots of \( p_3(x) = x^3 - 1 \)
3 Find all the real and complex roots of \( r_3(x) = x^3 + 1 \)
4 Find all the real and complex roots of \( q_3(x) = x^3 - 8 \)
5* Find all the real and complex roots of \( p_6(x) = x^6 - 1 \). What is the relationship between the roots of \( p_6(x) \) and \( p_3(x) \)?
6* What are the zeros of \( r_4(x) = p_4(ix) \)? How do they compare to the zeros of \( p_4(x) \)? (Hint: think of this as a “horizontal” stretch.)

D Radicals: This section will be substantially less strange than the previous one. We are simply evaluating roots.

1 \( \sqrt[3]{1512} \)
2 \( \sqrt[4]{72x^5y^9} \)
3 \( \sqrt[3]{160x^7} \)
4* \( \sqrt[4]{x^4 - 2x^3 - 12x^2 - 14x - 5} \)
5* \( \sqrt[16]{16x^4 - 200x^2 + 625} \)
6 \( \sqrt[4]{1800y^3x^2} \)
7 \( \sqrt[5]{256y^3x^2z^3} \)
8 \( \sqrt[5]{686x^9\alpha^7} \)
9 \( \sqrt[5]{3125x^8\beta^7} \)
10 \( \sqrt[12]{216\sigma^6\psi^{12}} \)
11 \( \sqrt[7]{-24\lambda^7} \)
A “Simplifying” Radicals This section will focus on basic operations using radicals. The goal here is to have the result have the smallest amount possible left inside the radical. As in $\sqrt{8} = 2\sqrt{2}$.

1. $\sqrt{40} - \sqrt{1000}$
2. $\sqrt{81} + \sqrt{16}$
3. $\sqrt{135} + \sqrt{40} + \sqrt{100}$
4. $\sqrt{-32} + \sqrt{-1}$
5. $3\sqrt{x^2y^4} + xy \sqrt{27x^3y}$
6. $\sqrt{16x^4} + \sqrt{x^6}$
7* $\sqrt{x^{3n}(x-5)^{n+1}(y-z)^{2n-1}}$ Assume that $n$ is a whole number at least 1.

B Radical Functions For the following set of problems identify the domain of the function.

1. $f(x) = 5\sqrt{x + 10}$
2. $g(x) = -2\sqrt{-2x + 5}$
3. $h(x) = \sqrt{x - 7}$
4. $i(x) = 13\sqrt{x + 9} + 5\sqrt{-x - 7}$
5. $j(x) = \sqrt{x^2 - 4x + 4}$
6. $k(x) = \sqrt{x^2 - 16}$
7* $l(x) = \sqrt{-x^2 - x - 1}$

C Radical Equations. For the following set of problems, the goal is to find the values for the variable that make the equation true. In other words, “solve” the equation. Then figure out which solutions are extraneous.

1. $\sqrt{25x + 125} = 10$
2. $\sqrt{25x + 125} = -10$
3. $\sqrt{25x + 125} = 10x$
4. $x + 1 = 2\sqrt{x^2 + 4x + 2}$
5* $x + 1 = \sqrt{x^2 + 2x + 1}$
6. $2x = \sqrt{9x^2 - 13x - 6}$
\[ \begin{align*}
7 \sqrt[3]{(x + 5)^2} &= 4 \\
8 \sqrt[2]{2x + 5} &= \sqrt{x - 2} + 2
\end{align*} \]

D Factoring with Complex Numbers. Factor the following polynomials completely over the complex numbers. To make this a little more easy you will be given one of the complex roots each time.

1. \[ x^4 - 2x^3 - 2x^2 + 8x - 8 \] one zero is \[ x = 1 + i \]
2. \[ 4x^4 + 8x^3 + 8x^2 + 8x + 4 \] one zero is \[ x = \frac{-5 + 3i}{3 + 5i} \]
3. \[ x^6 + x^4 - 8x^3 - 8x \] two zeros are \[ x = \frac{-5 - 2i}{2 - 5i}, \frac{-4i}{3 + i} \]