MAC 2233, Survey of Calculus, Exam 3 Review
This exam covers lectures 22 – 29,

This review includes typical exam problems. It is not designed to be comprehensive, but to be representative of topics covered on the exam. Be sure to work Exam 3 in the binder from Target Copy, and Exam 3 from fall available from the Broward Teaching Center.

1. Find the derivative of each of the following:
   (a) \( f(x) = 3^{2x-1} \)  
   (b) \( f(x) = \log_4(x^2 - x) \)

2. Find the slope of the tangent line to \( f(x) = \ln|2 - \ln x| \) at \( x = e \).

3. Find the slope of the tangent line to the curve given by \( \sqrt{3x - y} - e^{x+y} = 1 + \ln x \) at \((1, -1)\).

4. Find the first three derivatives of \( f(x) = \ln(x^2 + 1) \).

5. Find each value of \( x \) at which \( f(x) = \ln\left(\frac{2x - 6}{\sqrt{x^2 + 3}}\right) \) has a horizontal tangent line.

6. Find \( f'(x) \) if \( f(x) = \ln\frac{e^{x-3}\sqrt[3]{6 + 3x}}{(3x + 1)^2} \).

7. Use Logarithmic Differentiation to find the slope of the tangent line to \( f(x) = x\sqrt{x} \) at \( x = 4 \).

8. Find each value at which \( f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2 \) has a relative maximum or minimum. Find the absolute extrema of \( f(x) \) on \([-2, 1]\).
9. Find all critical numbers and relative extrema of \( g(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2} \).

10. Let \( f(x) = \frac{\sqrt[3]{3x} - 2}{x} \).

(a) Find \( f'(x) \) and write as a single fraction.
(b) Find the equation of each horizontal and vertical tangent line of \( f(x) \).
(c) Find each \( x \)-value at which \( f(x) \) has a critical number.
(d) Find the relative extreme values of \( f(x) \).

11. The cost function for a product is \( C(x) = 1.25x^2 + 25x + 8000 \).

(a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.
(b) Find each interval on which average cost is increasing and decreasing. For what production level \( x \) is average cost minimized?

12. The demand function for a certain product is given by
\[ p(x) = -0.02x + 400, \quad 0 \leq x \leq 20,000, \] where \( p \) is the unit price when \( x \) items are sold. The cost function for the product is
\[ C(x) = 100x + 300,000. \]

(a) Find the marginal profit of the product when \( x = 2000 \).
(b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (b).
(c) Find each interval on which the profit function is increasing and decreasing. Remember that \( 0 \leq x \leq 20,000 \). How many items should be sold to maximize profit? At what price?

13. Find all relative extrema of \( f(x) = 2x^{5/3} - 5x^{2/3} \). Then find the absolute extrema of \( f(x) \) on \([-8, 0]\). Compare the two methods.
14. Find the absolute maximum and minimum values of \( f(x) = e^{x^3 - 12x} \) on \([0, 3]\).

15. Find the maximum and minimum values of \( f(x) = x^2 - 8 \ln x \) on \([1, e]\).

16. The position (in centimeters) of a particle moving in a straight line at time \( t \) (in seconds) is given by \( s(t) = t^3 - 6t^2 + 9t \) for \( 0 \leq t \leq 6 \).

   (a) Find the velocity function \( v(t) \).

   (b) At what time(s) is the particle at rest?

   (c) For what time interval(s) over the first six seconds is the particle traveling in a positive direction?

   (d) Find the average velocity from \( t = 0 \) to \( t = 4 \) seconds.

   (e) What is the acceleration of the particle after \( 3/2 \) second? Include units in your answer.

   (f) Find each interval on which the particle is (1) speeding up and (2) slowing down

17. Find each interval on which \( f(x) = e^{1-x^2} \) is concave up and down, and find each inflection point of the graph of \( f \).

18. Find all intervals on which the graph of \( f(x) = \frac{x^4}{4} + 2x^3 + \frac{9}{2}x^2 + 8 \) is both decreasing and concave up.

19. Find each interval on which \( f(x) = \ln x + \frac{1}{x} \) is both increasing and concave down. Find each inflection point.

20. Suppose that \( f(x) \) has horizontal tangent lines at \( x = -2, x = 1 \) and \( x = 5 \). If \( f''(x) > 0 \) on intervals \((-\infty, 0)\) and \((2, \infty)\) and \( f''(x) < 0 \) on the interval \((0, 2)\), find the \( x \)-values at which \( f(x) \) has relative extrema. Assume that \( f \) and \( f' \) are continuous on \((-\infty, \infty)\) and use the Second Derivative test.
21. Given the graph of the derivative $f'(x)$, find each interval on which the function $f(x)$ is increasing and decreasing, and find the $x$-coordinate of each point at which $f(x)$ has a local maximum or minimum value. Find each interval on which $f(x)$ is concave up and down, and the $x$-coordinate of each inflection point. Assume that the domain of $f(x)$ is $(-\infty, \infty)$.

![Graph of $f'(x)$](image)

$y = f'(x)$

22. Suppose that $f'(x) = 15x^4 - 15x^2$. Find each $x$-value at which the function $f(x)$ has relative extrema. Find the $x$-coordinate of each inflection point. Sketch a possible graph of $f(x)$ if $f(x)$ passes through the origin.

23. A drug that stimulates reproduction is introduced into a population of viruses. That population can be modeled by the function $P(t) = 30t^2 - t^3 + 200$, $0 \leq t \leq 30$, where $P(t)$ is the population after $t$ minutes.

(a) At what time does the population reach its maximum? What is the maximum population?

(b) At what time is the rate of growth of the population maximized?
24. A farmer wishes to fence an area next to his barn. He needs a wire fence that costs $1 per linear foot in front of the barn and wooden fencing that costs $2 per foot on the other sides. Find the lengths \( x \) (sides perpendicular to the barn) and \( y \) (side across from the barn) so that he can enclose the maximum area if his budget for materials is $4400.

25. Frye’s Electronics has started selling a new video game. In one of its Dallas stores, an average of 50 games sell per month at the regular price of $40. The manager of the department has observed that when the video game is put on sale, an average of 5 more games will sell for each $2 price decrease. If each video game costs the store $24 and there are fixed costs of $5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function \( p(x) \) is linear.

26. The revenue \( R(x) \) generated from sales of a certain product is related to the amount of money spent on advertising according to the model 
\[
R(x) = \frac{1}{10,000}(600x^2 - x^3), \quad 0 \leq x \leq 600,
\]
where \( x \) and \( R(x) \) are measured in thousands of dollars. Find each interval over which \( R(x) \) is increasing. For the interval on which \( R(x) \) is increasing, find the point of diminishing returns. Why is it significant?

27. Consider the function \( f(x) = x^{1/3}(x + 3) \) and its first two derivatives,
\[
f'(x) = \frac{4x + 3}{3x^{2/3}}, \quad f''(x) = \frac{4x - 6}{9x^{5/3}}.
\]
Find all intercepts, asymptotes, relative extrema and inflection points. Sketch the graph of \( f(x) \).

28. Sketch the graph of \( f(x) = \frac{x^3}{1 - x^2} \) if \( f'(x) = \frac{x^2(3 - x^2)}{(1 - x^2)^2} \) and 
\[
f''(x) = \frac{2x(x^2 + 3)}{(1 - x^2)^3}.
\]
29. Given the graph of the derivative $f'(x)$, find a possible graph of the function $f(x)$. Assume that $f(-1) = -2$, $f(0) = 0$, $f(1) = 1$, $f(2) = 3$ and $f(3) = 5$, and that $f(x)$ is a continuous function. Be sure to find all extrema and inflection points.