This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:

You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/
1. Find the average rate of change in the area of a circle as its radius changes from
   (a) 2 to 3 cm.
   (b) 2 to 2.1 cm.
   (c) 2.5 to 2 cm.

2. Two resistors with resistance \( R_1 \) and \( R_2 \) (measured in Ohms, denoted \( \Omega \)) are wired in parallel.
   The net resistance, \( R \), of the two obeys the rule
   \[
   \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.
   \]
   Suppose \( R_1 \) and \( R_2 \) are increasing at rates 0.3\( \Omega \)/s and 0.2\( \Omega \)/s respectively. How fast is \( R \)
   changing at the moment when \( R_1 = 80\Omega \) and \( R_2 = 100\Omega \)?

3. A kite flying 100 feet above the ground moves horizontally at a speed of 8 feet per second. At
   what rate is the angle between the string and the horizontal changing when 200 feet of string
   has been let out?

4. Use linear approximation to estimate the following numbers. \(^{Hint:}\) interpret the numbers as values of a differentiable function.
   (a) \( \sqrt{143.9} \)
   (b) \( (2018.1)^{-1} \)
   (c) \( (2018.1)^5 \)

5. Find the linearization for each of the functions at the prescribed place.
   (a) \( f(x) = \cos(x) \) at \( x = \pi/4 \).
   (b) \( g(x) = \ln(x - 1) \) at \( x = e \).
   (c) \( h(x) = \frac{1}{x^2} \) at \( x = 2 \).

6. Create a list of critical numbers for each function below.
   (a) \( f(x) = \sqrt{x^2(2x - 1)} \)
   (b) \( g(t) = 6t - 4 \cos(3t) \)
   (c) \( h(x) = 10xe^{3-x^2} \)

7. Locate and describe any local extrema for the functions from the previous problem.
8. Determine if the functions satisfy the hypotheses of the Mean Value Theorem on the intervals specified. For those that do, list the numbers guaranteed to exist by the theorem; for those that do not, explain why.

(a) \( f(x) = x^3 + 2x^2 - x \) on \([-1, 2]\)

(b) \( g(t) = 8t + e^{-3t} \) on \([-2, 3]\)

(c) \( h(x) = \begin{cases} x^2 & x \leq 2 \\ 4x - 4 & x > 2 \end{cases} \) on \([0, 4]\)

(d) \( X(s) = 1 - \frac{1}{s} \) on \([-1, 2]\)

9. Suppose \( f(x) \) is continuous on \([6, 15]\), differentiable on \((6, 15)\), \( f(6) = -2 \), and \( f'(x) \leq 10 \) for all \( x \). What is the largest possible value of \( f(15) \)? 

Hint: use MVT

10. Determine the intervals on which each of the following functions is increasing/decreasing.

(a) \( f(x) = x^{9/5} - x \)

(b) \( h(x) = 2\sin^2(x) - 2x \) restricted to \([0, \pi]\)

11. Find real numbers \( A \) and \( B \) such that the function \( f(x) = Ax^3 + Bx^2 + 1 \) has an inflection point at \((-1, 2)\).

12. Calculate the limits below, employing L’Hôpital’s rule as needed.

(a) \( \lim_{x \to \infty} \frac{e^{2018x} + 1}{e^{1729x}} \)

(b) \( \lim_{x \to 0} \frac{3^x - 1}{x} \)

(c) \( \lim_{x \to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin(x)} \)

(d) \( \lim_{t \to \infty} t^2 e^{-t} \)

(e) \( \lim_{x \to 0^+} x^x \)

(f) \( \lim_{x \to \infty} \sqrt{x^2 + 1} - x + 1 \)

13. Evaluate the limits below. If the limit does not exist, indicate why.

(a) \( \lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} \)

(b) \( \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right) \)

(c) \( \lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4} \)
14. Evaluate the limits below. If the limit does not exist, indicate why.

(a) \( \lim_{x \to \infty} \frac{7x^3 + x^2 - 5}{x^3 - 2x + 1} \)
(b) \( \lim_{x \to \infty} \frac{4x^5}{3 - 2x^5} \)
(c) \( \lim_{x \to \infty} \frac{x - 3}{x^2 + 2x + 1} \)
(d) \( \lim_{x \to \infty} \frac{2x + 1}{3x - 5} \)
(e) \( \lim_{x \to \infty} e^{\tan^{-1}(x)} \)

15. Let \( f(x) = x(6 - x)^{2/3} \).

(a) Find any zeros for \( f(x) \), and determine where \( f \) is positive/negative.
(b) Determine the function’s end behavior.
(c) Determine where \( f \) is increasing/decreasing, and find any local extrema.
(d) Determine where \( f \) is concave up/concave down, and find any inflection points.
(e) Sketch a graph of \( y = f(x) \), labeling all of the features discussed above.

16. Determine all extrema, local and absolute.

(a) \( f(x) = 2x^3 + 3x^2 - 12x + 4 \) on the interval \([-4, 2]\).
(b) \( f(x) = 2x^3 + 3x^2 - 12x + 4 \) on the interval \([0, 2]\).
(c) \( g(t) = 2000 - 10te^{5-t^2/8} \) on the interval \([0, 10]\).
1) A business is currently selling 1000 novelty wind chimes for $100 each. It is estimated that for each decrease in price of $20, the monthly sales will increase by 200 chimes.
   a) What is the marginal revenue when production is at 1100 chimes?
   b) Suppose a company is increasing the production of chimes by 15 chimes per month. Find the rate at which revenue is changing with respect to time when production is at 500 chimes. Include units.

2) A young gentleman is standing 10 feet from a building where his significant other is throwing his stuff out of their apartment window 20 feet above the main apartment entrance which is horizontally lined up with the gentleman’s line of site if he looks straight ahead. However, the gentleman can’t take his eyes off of the falling objects... especially the T.V. ... and his line of sight makes an angle, $\theta$, relative to him looking at the apartment entrance. How fast is this angle decreasing when his T.V. is half-way down assuming its falling at a rate of 30 ft/second at that position?

3) Gravel is being dumped from a conveyor belt at a rate of 30 ft$^3$/min. As it falls from the conveyor belt, it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

4) How many critical values does the function $f(x) = \frac{e^x}{1-x+x^2}$ have, and are they local minima or maxima (local extrema), or neither?

5) Find the absolute minimum and maximum values for each of the following functions.
   a) $f(x) = \ln(x^2 + x + 1)$ over the interval $[-1,1]$
   b) $f(t) = t\sqrt{4-t^2}$ over the interval $[-1,2]$
   c) $f(\theta) = \theta - 2 \cos \theta$ over the interval $[-\pi, \pi]$

6) Find all numbers, $c$, that satisfy the conclusion of Rolle’s Theorem [0,2π] for the function $\sin x + \cos x$.

7) Consider the graph of $f'(x)$ which is given below for the continuous function $f(x)$.

   ![Graph of f'(x)](image)

   a) Where does $f(x)$ have critical numbers?
   b) On what intervals is $f(x)$ increasing/decreasing?
   c) On what intervals is $f(x)$ concave up/down?
   d) Sketch a possible graph of $f(x)$.

8) Find the critical numbers of the following functions. Identify the intervals where the function is increasing and decreasing by using the first derivative test.
   a) $f(x) = x^2 e^{-3x}$
   b) $g(x) = x^{-2} \ln(x)$
   c) $k(\theta) = 2 \cos(\theta) + \sin^2(\theta)$
9) Approximate 1.986 using differentials.

10) Consider the following functions,

\[ f(x) = \frac{x^2}{(x - 2)^2}, \quad g(x) = x^3 + 2x^{\frac{1}{3}} \]

List the locations of...

a) The vertical and horizontal asymptotes
b) The critical numbers, and the intervals where the function is increasing, decreasing, concave up, and concave down.
c) Local Minimum, local Maximum, and Points of Inflection
d) Sketch a graph of \( f(x) \).

11) How many inflection points does the following function have?

\[ g(x) = 3x^5 - 10x^4 + 10x^3 \]

12) Let \( p(x) = -0.3x + 20 \) and \( C(x) = 5x + 12 \) be the demand and cost functions, respectively, which model a particular company’s new product when production is between 0 and 100 units.

a) Find the profit function, \( P(x) \).

b) Find the profit at 30 units.

c) Use differentials to estimate the change in the company’s profit between producing 30 units to 32 units.

d) Approximately, what is the company profit when they produce 32 units?

e) What is the company’s actual profit when they produce 32 units?

f) For which range of production levels is the company’s Profit increasing and decreasing?

g) What prices should the company charge in order to reach the maximum profit

13) Suppose \( g(2) = 3 \) and \( |g'(x)| \leq 4 \), what is largest and smallest possible value of \( g(-1) \)?

14) Use the linearization of the function \( \sqrt{3x - 4} \) at \( a = 4 \) to approximate the value of \( \sqrt{8.012} \).

15) Consider the function \( f(x) = x^3 + x - 1 \)

a) Find all the numbers, \( c \), that satisfy the conclusion of the Mean Value Theorem on \([0, 2]\).

b) Use information about the derivative of \( f \) and to show that \( f(x) \) has only one real root.

16) Find the limit. You may use L’Hospital’s Rule if it applies. If it does not apply, state why.

a) \( \lim_{x \to 0} \frac{e^x}{x^2} \)

b) \( \lim_{x \to 0} \frac{x + \sin x}{x + \cos x} \)

c) \( \lim_{x \to 0} \frac{x^2}{e^{x-1}-x} \)

d) \( \lim_{x \to 0} \frac{\pi}{4} (1 - \tan x) \sec x \)

e) \( \lim_{x \to 0} (\sin x)^{\tan x} \)