MAC 2233
Fall 2014
Exam 2A

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 2, 1.
   1   ○  3  4  5  6  7  8  9  0
   ○  2  3  4  5  6  7  8  9  0

D. At the top right of your answer sheet, for “Test Form Code”, encode A.
   ○  B  C  D  E

E. 1) This test consists of 15 multiple choice questions, ranging from five points to one point in value, plus two sheets (four pages) of free response questions worth 30 points. The test is counted out of 80 points, and there are six bonus points available on the multiple choice.

   MAKE SURE YOUR EXAM IS COMPLETE WITH ALL QUESTIONS.

   2) The time allowed is 90 minutes.

   3) You may write on the test, since you will take it with you after the exam.

   4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR SCANTRON COVERED AT ALL TIMES.

G. REMEMBER that you have agreed to the UF Honor Pledge: “On my honor, I have neither given nor received unauthorized aid in doing this exam.”

H. When you are finished:

   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.

   2) You must turn in your scantron and tearoff sheet to the proctor of the exam. Be prepared to show a picture ID with a legible signature.

   3) Answers will be posted on the MAC 2233 homepage after the exam. Your score will be posted in Sakai (E-Learning) within one week of the exam.
NOTE: Be sure to bubble the answers to questions 1–15 on your scantron.

Questions 1 - 8 are worth 5 points each.

1. Evaluate the limit: \( \lim_{x \to -3} \frac{1}{x} - \frac{1}{x^2 - 12} \).

   a. \( \frac{-7}{9} \)   b. \(-7\)   c. 0   d. \( \frac{-1}{9} \)   e. The limit does not exist.

2. The horizontal tangent lines of \( f(x) = \frac{1}{4}x^4 + \frac{4}{3}x^3 - \frac{5}{2}x^2 + 2 \) occur at which of the following \( x \)-values?

   a. \( x = -1 \) and \( x = 5 \)   b. \( x = -5 \) and \( x = 1 \)   c. \( x = 0 \) only
   d. \( x = -1, x = 0 \) and \( x = 5 \)   e. \( x = -5, x = 0 \) and \( x = 1 \)

3. Evaluate \( \lim_{x \to 1} f(x) \) given that \( f(x) = \begin{cases} \frac{1 - x}{\sqrt{x^2 + 3} - 2} & x \neq 1 \\ 3 & x = 1 \end{cases} \).

   a. \( \frac{-1}{2} \)   b. 3   c. \(-2\)   d. \(-4\)   e. The limit does not exist.

4. Find the value of \( k \) so that the tangent line to \( f(x) = 4\sqrt{x} + kx^2 - 3 \) at \( x = 4 \) is perpendicular to the line \( 3y + x = 6 \). Hint: what is the slope of the tangent line to \( f(x) \) at \( x = 4 \)?

   a. \( k = -\frac{1}{24} \)   b. \( k = \frac{1}{2} \)   c. \( k = \frac{1}{4} \)   d. \( k = -\frac{3}{4} \)   e. \( k = \frac{3}{2} \)
5. Let \( f(x) = \begin{cases} 
3x + 1 & x < -1 \\
\frac{2}{x} & -1 < x \leq 2 \\
x^2 - 2 & x > 2 
\end{cases} \)

Which of the following statements is/are true about \( f(x) \)?

P. \( f(x) \) has a jump discontinuity at \( x = -1 \).
Q. \( f(x) \) has a nonremovable discontinuity at \( x = 0 \).
R. The graph of \( f(x) \) can be made continuous at \( x = -1 \) by defining \( f(-1) = -2 \).
S. \( \lim_{x \to 2} f(x) = f(2) \).

a. P, Q and S  
b. Q and R only  
c. R only  
d. P, Q and R  
e. P and S only

6. Let \( p = \lim_{x \to -\infty} \frac{5x - 1}{|x| - 2} \) and let \( q = \lim_{x \to +\infty} \frac{2}{3 + e^{-x}} \). Then

a. \( p = 5 \) and \( q = \frac{2}{3} \)

b. \( p = -5 \) and \( q = 0 \)

c. \( p = \frac{1}{2} \) and \( q = 0 \)

d. \( p = -5 \) and \( q = \frac{2}{3} \)

6. Let \( p = 5 \) and \( q = +\infty \)

3A
11. Find $f'(x)$ if $f(x) = x^2(x - 3)^4$.

a. $f'(x) = 8x(x - 3)^3$
b. $f(x) = 6x(x - 3)^3(x - 1)$
c. $f(x) = 2x(x - 3)^3(x + 3)$
d. $f(x) = (x - 3)^3(4x^2 + 1)$
e. $f(x) = 4x(x - 3)^3(2x + 1)$

The following problems are worth 2 points each.

12. According to the Intermediate Value Theorem, the function $f(x) = \frac{x^2 - 2}{x^2 + 1}$ has at least one zero (the equation $f(x) = 0$ has at least one solution) on which of the following intervals?

a. $(-1, 0)$  
 b. $(-3, -2)$  
 c. $(2, 3)$  
 d. $(0, 1)$  
 e. $(-2, -1)$

13. (Bonus!) The demand and cost functions for a given product are $p = 120 - 0.5x$ and $C(x) = x^2 + 20x + 15$, where $x$ is the number of units sold, $0 \leq x \leq 240$. Find the value of $x$ for which the marginal profit is $10 per unit.

a. $x = 15$  
 b. $x = 10$  
 c. $x = 30$  
 d. $x = 25$  
 e. $x = 20$

Be sure to work the bonus problems on the next page.
7. Write the equation of the tangent line to \( f(x) = \sqrt{x^2 + 2x + 4} \) at \( x = -2 \).

   a. \( y = -\frac{1}{2}x + 1 \)  
   b. \( y = \frac{1}{4}x + \frac{5}{2} \)  
   c. \( y = -x \)  
   d. \( y = \frac{1}{4}x + \frac{1}{2} \)  
   e. \( y = -\frac{1}{2}x - 3 \)

8. The position function of a vehicle moving along a straight track is given by \( s(t) = \frac{3t^2}{2t - 1} \), where \( s(t) \) is the vehicle's distance in meters from an observer \( t \) seconds after passing in front of him. Find the velocity of the vehicle after 2 seconds.

   a. \( \frac{8}{3} \) m/sec  
   b. 4 m/sec  
   c. \( \frac{5}{9} \) m/sec  
   d. 12 m/sec  
   e. \( \frac{4}{3} \) m/sec

The following problems are worth 3 points each.

9. Find each value of \( s \) so that \( f(x) = \begin{cases} 3x + s^2 & x < -1 \\ 3s - x & x \geq -1 \end{cases} \) is continuous for all real numbers.

   a. \( s = -2 \) and \( s = 2 \)  
   b. \( s = -3 \) and \( s = 1 \)  
   c. \( s = 4 \) only  
   d. \( s = -1 \) and \( s = 4 \)  
   e. There are no such values of \( s \).

10. Find the slope of the tangent line to \( f(x) = \frac{2x^{3/2} - 3\sqrt{x} + 4x}{\sqrt{x}} \) at \( x = 4 \).

    a. 13  
    b. 3  
    c. \( \frac{9}{4} \)  
    d. 21  
    e. 4

4A
Bonus! (1 point each)

14. Suppose that \( f(x) = \frac{p(x)}{q(x)} \) where \( p \) is a polynomial of degree \( m \geq 1 \) and \( q \) is a nonzero polynomial of degree \( n \). If \( f(x) \) has no horizontal asymptote, then which must be true?

   a. \( m = n \)  
   b. \( m > n \)  
   c. \( m < n \)

Indicate if each of the following statements is true or false.

15. The function \( f(x) = |x - 1| \) is continuous but not differentiable at \( x = 1 \).

   a. True  
   b. False

16. If \( H(x) = x^2 \cdot f(x) \), then \( H'(x) = 2x \cdot f'(x) \).

   a. True  
   b. False
1. Let \( f(x) = \frac{1}{x+3} \).

(a) Find the average rate of change of \( f(x) \) as \( x \) changes from \( x = a \) to \( x = a + h \), \( h \neq 0 \).

(b) Find the instantaneous rate of change of \( f(x) \) with respect to \( x \) when \( x = a \).

(c) Find the derivative \( f'(x) \).

\[ f'(x) = \]
2. Let \( f(x) = \frac{2x^2 - 5x + 2}{x^2 - 4} \).

Find the following for \( f(x) \). If none, write "none".

(a) Evaluate the following limits:

(1) \( \lim_{x \to 2^-} f(x) = \) \_

(2) \( \lim_{x \to 2^+} f(x) = \) \_

(3) \( \lim_{x \to \infty} f(x) = \) \_

(b) Find each vertical and horizontal asymptote of \( f(x) \).

(c) List each \( x \)-value at which \( f(x) \) has a discontinuity. If the discontinuity is removable, state how you could define \( f(x) \) to make \( f(x) \) continuous at that \( x \)-value.

3. The total cost of producing \( x \) units of a product can be modeled by the function \( C(x) = 600 + 40x + 0.02x^2 \), \( 0 \leq x \leq 200 \). Consider the **average cost** per unit.

(a) The average cost function \( \overline{C}(x) = \) \_

(b) Find the **marginal average cost** when \( x = 100 \). Be sure to interpret your answer.
4. Let \( f(x) = \begin{cases} 
2x - 1 & x \leq 1 \\
x^2 & x > 1 
\end{cases} \).

(a) Find \( f(1) \).

(b) Find \( f'(1) \) if possible. Use the limit definition of derivative at a point

\[
f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}.
\]

1) \( \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \) ____________

2) \( \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \) ____________

\( f'(1) = \) _______________ (if not differentiable, so state)

5. A function \( y = f(x) \) is given below. Sketch a possible graph of the derivative function \( y = f'(x) \) on the given axes.
6. The number of bacteria $N(t)$ in a certain culture $t$ minutes after a chemical bactericide is introduced is modeled by the function $N(t) = \frac{30}{t^2 + 1} + 8$, where $N$ is measured in thousands. Answer the following, including units.

(a) What was the population of bacteria in the culture when the bactericide was first introduced?

____________________ bacteria

(b) What is the average rate of change in the number of bacteria during the first three minutes after the bactericide is introduced ($t = 0$ to $t = 3$)? Interpret your results including units in your answer.

(c) At what rate is the number of bacteria changing after three minutes (when $t = 3$)? Interpret your results.

(d) What will be the least number of bacteria observers can expect to find in the culture as time continues? Hint: find $\lim_{t \to \infty} N(t)$.

____________________ bacteria

10A
MAC 2233  
Fall 2014  
Exam 2B  

A. Sign your scantron on the back at the bottom in ink.  

B. In pencil, write and encode in the spaces indicated:  
   1) Name (last name, first initial, middle initial)  
   2) UF ID Number  
   3) Section Number  

C. Under “special codes”, code in the test ID number 2, 2.  
   1  ⋄  3  4  5  6  7  8  9  0  
   1  ⋄  3  4  5  6  7  8  9  0  

D. At the top right of your answer sheet, for “Test Form Code”, encode B.  
   □  □  □  □  □  

E.  1) This test consists of 15 multiple choice questions, ranging from five points to one 
    point in value, plus two sheets (four pages) of free response questions worth 30 
    points. The test is counted out of 80 points, and there are six bonus points available 
    on the multiple choice.  

    MAKE SURE YOUR EXAM IS COMPLETE WITH ALL QUESTIONS.  

    2) The time allowed is 90 minutes.  
    3) You may write on the test, since you will take it with you after the exam.  
    4) Raise your hand if you need more scratch paper or if you have a problem with your 
    test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE 
    TEST.  

F. KEEP YOUR SCANTRON COVERED AT ALL TIMES.  

G. REMEMBER that you have agreed to the UF Honor Pledge: "On my honor, I have 
    neither given nor received unauthorized aid in doing this exam."  

H. When you are finished:  
   1) Before turning in your test, check for transcribing errors. Any mistakes you 
      leave in are there to stay.  
   2) You must turn in your scantron and tearoff sheet to the proctor of the exam. Be 
      prepared to show a picture ID with a legible signature.  
   3) Answers will be posted on the MAC 2233 homepage after the exam. Your score will 
      be posted in Sakai (E-Learning) within one week of the exam.
NOTE: Be sure to bubble the answers to questions 1–15 on your scantron.

Questions 1 - 8 are worth 5 points each.

1. Evaluate the limit: \( \lim_{{x \to -2}} \frac{1}{x} - \frac{1}{x^2 - 6}. \)
   
   a. \(-5\)    b. \(0\)    c. \(-\frac{1}{4}\)    d. \(-\frac{5}{4}\)    e. The limit does not exist.

2. Let \( p = \lim_{{x \to +\infty}} \frac{5}{4 + e^{-x}} \) and let \( q = \lim_{{x \to -\infty}} \frac{3x + 2}{|x| - 1} \). Then
   
   a. \( p = \frac{5}{4} \) and \( q = 3 \)
   
   b. \( p = 0 \) and \( q = -3 \)
   
   c. \( p = \frac{5}{4} \) and \( q = -3 \)
   
   d. \( p = +\infty \) and \( q = 3 \)
   
   e. \( p = 0 \) and \( q = -2 \)

3. The horizontal tangent lines of \( f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 - \frac{5}{2}x^2 + 2 \) occur at which of the following \( x \)-values?
   
   a. \( x = -1, x = 0 \) and \( x = 5 \)    b. \( x = -5, x = 0 \) and \( x = 1 \)
   
   c. \( x = -1 \) and \( x = 5 \)    d. \( x = 0 \) only    e. \( x = -5 \) and \( x = 1 \)
4. Evaluate \( \lim_{x \to 2} f(x) \) given that 
\[
\begin{cases} 
\frac{2-x}{\sqrt{x^2+5} - 3} & x \neq 2 \\
1 & x = 2
\end{cases}
\]

a. 1  
b. -3  
c. \frac{1}{4}  
d. -\frac{3}{2}  
e. The limit does not exist.

5. Write the equation of the tangent line to \( f(x) = \sqrt{x^2 + 2x + 4} \) at \( x = -2 \).

a. \( y = -\frac{1}{2}x - 3 \)  
b. \( y = -x \)  
c. \( y = \frac{1}{4}x + \frac{5}{2} \)

d. \( y = \frac{1}{4}x + \frac{1}{2} \)  
e. \( y = -\frac{1}{2}x + 1 \)

6. Let 
\[
\begin{cases} 
2x + 1 & x < -2 \\
\frac{6}{x} & -2 < x \leq 2 \\
x^2 - 2 & x > 2
\end{cases}
\]

Which of the following statements is/are true about \( f(x) \)?

P. \( f(x) \) has a jump discontinuity at \( x = -2 \).
Q. \( \lim_{x \to 2} f(x) = f(2) \).
R. \( f(x) \) has a nonremovable discontinuity at \( x = 0 \).
S. The graph of \( f(x) \) can be made continuous at \( x = -2 \) by defining \( f(-2) = -3 \).

da. P, R and S  
b. R and S only  
c. S only

d. P and Q only  
e. P, Q and R
7. The position function of a vehicle moving along a straight track is given by \( s(t) = \frac{5t^2}{3t - 1} \), where \( s(t) \) is the vehicle’s distance in meters from an observer \( t \) seconds after passing in front of him. Find the velocity of the vehicle after 2 seconds.

a. \( \frac{16}{5} \) m/sec  
b. 15 m/sec  
c. \( \frac{8}{5} \) m/sec  
d. \( \frac{10}{3} \) m/sec  
e. 4 m/sec

8. Find the value of \( k \) so that the tangent line to \( f(x) = 4\sqrt{x} + kx^2 - 5 \) at \( x = 4 \) is perpendicular to the line \( 3y + x = 9 \). Hint: what is the slope of the tangent line to \( f(x) \) at \( x = 4 \)?

a. \( k = \frac{1}{4} \)  
b. \( k = -\frac{3}{4} \)  
c. \( k = \frac{1}{2} \)  
d. \( k = \frac{3}{2} \)  
e. \( k = -\frac{1}{6} \)

The following problems are worth 3 points each.

9. Find the slope of the tangent line to \( f(x) = \frac{2x^{3/2} - 3\sqrt{x} + 4x}{\sqrt{x}} \) at \( x = 4 \).

a. 21  
b. 4  
c. 13  
d. \( \frac{9}{4} \)  
e. 3

10. Find \( f'(x) \) if \( f(x) = x^2(x - 3)^4 \).

a. \( f(x) = (x - 3)^3(4x^2 + 1) \)  
b. \( f(x) = 4x(x - 3)^3(2x + 1) \)  
c. \( f'(x) = 8x(x - 3)^3 \)  
d. \( f(x) = 6x(x - 3)^3(x - 1) \)  
e. \( f(x) = 2x(x - 3)^3(x + 3) \)
11. Find each value of $s$ so that $f(x) = \begin{cases} 4x + s^2 & x < -2 \\ 3s - x & x \geq -2 \end{cases}$ is continuous for all real numbers.

a. $s = -2$ and $s = 2$  
   b. $s = -1$ and $s = 4$  
   c. $s = 5$ only  

    d. $s = -2$ and $s = 5$  
    e. There are no such values of $s$.

The following problems are worth 2 points each.

12. According to the Intermediate Value Theorem, the function

$$f(x) = \frac{x^2 - 3}{x^2 + 2}$$

has at least one zero (the equation $f(x) = 0$ has at least one solution) on which of the following intervals?

a. $(-3, -2)$  
   b. $(2, 3)$  
   c. $(-2, -1)$  
   d. $(0, 1)$  
   e. $(-1, 0)$

13. (Bonus!) The demand and cost functions for a given product are

$p = 100 - 0.5x$ and $C(x) = x^2 + 40x + 10$, where $x$ is the number of units sold, $0 \leq x \leq 200$. Find the value of $x$ for which the marginal profit is $30 per unit.

a. $x = 20$  
   b. $x = 15$  
   c. $x = 30$  
   d. $x = 25$  
   e. $x = 10$

Be sure to work the bonus problems on the next page.
Bonus! (1 point each)

14. Suppose that \( f(x) = \frac{p(x)}{q(x)} \) where \( p \) is a polynomial of degree \( m \geq 1 \) and \( q \) is a nonzero polynomial of degree \( n \). If \( f(x) \) has no horizontal asymptote, then which must be true?

   a. \( m > n \)  
   b. \( m = n \)  
   c. \( m < n \)

Indicate if each of the following statements is true or false.

15. If \( H(x) = x^3 \cdot f(x) \), then \( H'(x) = 3x^2 \cdot f'(x) \).

   a. True  
   b. False

16. The function \( f(x) = |x+3| \) is continuous but not differentiable at \( x = -3 \).

   a. True  
   b. False
1. Let \( f(x) = \frac{1}{x-4} \).

(a) Find the average rate of change of \( f(x) \) as \( x \) changes from \( x = a \) to \( x = a + h \), \( h \neq 0 \).

(b) Find the instantaneous rate of change of \( f(x) \) with respect to \( x \) when \( x = a \).

(c) Find the derivative \( f'(x) \).

\[ f'(x) = \hspace{1cm} \]
2. The number of bacteria $N(t)$ in a certain culture $t$ minutes after a chemical bactericide is introduced is modeled by the function $N(t) = \frac{60}{t^2 + 1} + 5$, where $N$ is measured in hundreds. Answer the following, including units.

(a) What was the population of bacteria in the culture when the bactericide was first introduced?

__________________ bacteria

(b) What is the average rate of change in the number of bacteria during the first three minutes after the bactericide is introduced ($t = 0$ to $t = 3$)? Interpret your results including units in your answer.

(c) At what rate is the number of bacteria changing after three minutes (when $t = 3$)? Interpret your results.

(d) What will be the least number of bacteria observers can expect to find in the culture as time continues? Hint: find $\lim_{t \to \infty} N(t)$.

__________________ bacteria
3. Let \( f(x) = \begin{cases} x^2 & x \leq 2 \\ 4x - 4 & x > 2 \end{cases} \).

(a) Find \( f(2) \).

(b) Find \( f'(2) \) if possible. Use the limit definition of derivative at a point
\[
f'(1) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.
\]

1) \( \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \) ____________

2) \( \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \) ____________

\( f'(2) = \) ________________ (if not differentiable, so state)

4. A function \( y = f(x) \) is given below. Sketch a possible graph of the derivative function \( y = f'(x) \) on the given axes.
5. Let $f(x) = \frac{3x^2 - 5x - 2}{x^2 - 4}$.

Find the following for $f(x)$. If none, write "none".

(a) Evaluate the following limits:

$\lim_{x \to 2^-} f(x) = \underline{\hspace{2cm}}$ \hspace{1cm} \( \lim_{x \to 2^+} f(x) = \underline{\hspace{2cm}} \)

$\lim_{x \to +\infty} f(x) = \underline{\hspace{2cm}}$

(b) Find each vertical and horizontal asymptote of $f(x)$.

(c) List each $x$-value at which $f(x)$ has a discontinuity. If the discontinuity is removable, state how you could define $f(x)$ to make $f(x)$ continuous at that $x$-value.

6. The total cost of producing $x$ units of a product can be modeled by the function $C(x) = 800 + 50x - 0.02x^2$, $0 \leq x \leq 200$. Consider the average cost per unit.

(a) The average cost function $\bar{C}(x) = \underline{\hspace{12cm}}$.

(b) Find the marginal average cost when $x = 100$. Be sure to interpret your answer.