This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:

You can learn more about the services offered by the teaching center by visiting [https://teachingcenter.ufl.edu/](https://teachingcenter.ufl.edu/)
1. Investigate the convergence of the series \( \sum_{n=1}^{\infty} \left( \frac{2n+3}{3n+2} \right)^n \).

2. Suppose that \( \sum_{n=0}^{\infty} c_n(x + 1)^n \) converges for \( x = 1/2 \) and diverges for \( x = -4 \). Consider:

\[
A = \sum_{n=0}^{\infty} c_n(-5)^n \quad B = \sum_{n=0}^{\infty} c_n \quad C = \sum_{n=0}^{\infty} c_n 2^n
\]

What can be said about the convergence of series \( A \), \( B \), and \( C \)?

3. Investigate the convergence of \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{4n} \).

4. Use the fact that \( 4 \arctan(1) = \pi \), and \( \frac{1}{1 + x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \) to find a series representation for the number \( \pi \). [Hint: integrate]

5. Express \( \int_0^x \frac{e^t - 1 - t}{t^2} dt \) as a power series. Find the IOC.

6. Evaluate \( \int \frac{1}{\sqrt{3 - 2x - x^2}} \, dx \)

7. Find the sum of the series \( S = \sqrt{2} - \frac{\sqrt{2\pi^2}}{4^2 \cdot 2!} + \frac{\sqrt{2\pi^4}}{4^4 \cdot 4!} - \frac{\sqrt{2\pi^6}}{4^6 \cdot 6!} + \cdots \)

8. Evaluate the definite integral \( \int_0^1 \sqrt{4 - x^2} \, dx \).

9. Set up the partial fraction decomposition for \( \frac{1}{x^4 - 9x^2} \).
10. Complete the partial fraction decomposition from the previous problem and use it to evaluate
\[ \int \frac{1}{x^4 - 9x^2} \, dx. \]

11. Evaluate \( \int x^2 \sqrt{x^2 - 4} \, dx \)

12. Which of the following integrals converge?

I. \( \int_2^\infty \frac{1}{x^{\pi/3}} \, dx \)

II. \( \int_1^\infty \frac{e^t}{1 + e^{4t}} \, dt \)

III. \( \int_2^\infty \frac{1}{\ln(s)} \, ds \)

13. Find a closed form for the \( N^{th} \) partial sum of \( \sum_{n=2}^{\infty} \ln \left( \frac{n}{n+1} \right) \)

14. Sketch the polar curve \( r = 1 + \cos(\theta) \), and set up an integral to find the area above the horizontal axis.

15. Given \( r = 1 - 2 \cos(\theta) \) set up an integral for the area of the inner loop.

16. Set up an integral for the area inside \( r_2 \) and outside \( r_1 \) where \( r_1 = \sqrt{3} \sin(\theta) \) and \( r_2 = \cos(\theta) \).

17. Given \( x(t) = e^t + 5t \) and \( y(t) = 100 \), find the arclength from \( t = 0 \) to \( t = 5 \).

18. At what points (if any) does the parametric curve have horizontal or vertical tangent lines?

\[ x(t) = \cos^2(t) + \cos(t) \quad y(t) = \sin(t) \cos(t) + \sin(t) \]

19. Consider the solid obtained by rotating the region bounded by \( y = \ln(x) \), \( x = e \), and \( y = 0 \) about the \( x \)-axis. Set up two integrals for the volume of this solid. One using the disk/washer method, the other using cylindrical shells.
Fall 2018 Final Exam is roughly broken down as follows:

Integration unit     (3) (+1)
Convergence tests    (5)
Power Series         (3)
Parametric           (3)
Polar                (3)
Volume               (5)

Final contains: All multiple choices. 20 questions worth 3 points/ea and 2 extra credit questions worth 1.5 points/ea.

’Maclaurin Series’ sheet is provided on the final.
Power Series

1. Evaluate \( \int \frac{t}{1 - t^7} \, dt \) as an infinite series.

2. Suppose \( \sum c_n x^n \) converges at \( x = 6 \). Which below MUST be convergent?

\[ P: \sum c_n (-4)^n, \quad Q: \sum c_n (-6)^n \]

(a) P only
(b) Q only
(c) Both
(d) Neither
3. Suppose $\sum c_n x^n$ converges at $x = 6$. Which below is NOT possible?

P: $\sum c_n (-4)^n$, diverges
Q: $\sum c_n (-6)^n$ converges

(a) P only
(b) Q only
(c) Both
(d) Neither
4. Find a power series for $e^{2x}$, centered at 6.

(a) $\sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!}$

(b) $\sum_{n=0}^{\infty} e^6 \frac{(x - 6)^n}{n!}$

(c) $e^{12} \sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!}$

(d) $e^{12} \sum_{n=0}^{\infty} \frac{2^n(x - 6)^n}{n!}$
5. Find a power series for $f(x)$, centered at 0.

$$f(x) = \frac{x}{(1 + x)^2}$$
6. Find the sum of the series

\[ \sum_{n=2}^{\infty} \left( \frac{-2^{2n}}{(-9)^{n-1}} + \frac{(-1)^n}{(2n)!} \right) \]

(a) \( \frac{16}{13} + \cos 1 \)

(b) \( -\frac{16}{11} - \cos 1 \)

(c) \( \frac{16}{11} + e \)

(d) \( -\frac{16}{13} - \sin 1 \)

(e) \( \frac{11}{9} + \cos 1 \)
7. Find the value of $t$ for which the series \[ \sum_{n=0}^{\infty} \frac{(-1)^n t^{n+1}}{3^{2n}(2n + 1)} \]
converges, and find the function it converges to for these values of $t$.

(ans. \( f(t) = t \arctan \left( \frac{t}{3} \right), \, t \in [-3,3] \))
8. (TRUE/FALSE)

Let

\[ f(0) = 4, \quad f'(0) = 3, \]
\[ f''(0) = 3, \quad f'''(0) = 2. \]

The first 4 terms of the Maclaurin Series of \( f \) is

\[ f(x) = 4 + 3x + 3x^2 + 2x^3 \]

(a) TRUE

(b) FALSE
Parametric and Polar:

9. Find the length of the path over the given interval.

\[ c(t) = (t^3 + 8, t^2 + 3), \quad 0 \leq t \leq 1 \]

(a) \( (100 + \frac{4}{9})^{3/2} - (25 + \frac{4}{9})^{3/2} \)

(b) \( (100 + \frac{9}{4})^{3/2} - (25 + \frac{9}{4})^{3/2} \)

(c) \( (100 + \frac{4}{9})^{3/2} + (25 + \frac{4}{9})^{3/2} \)

(d) \( \frac{1}{27}(13^{3/2} - 4^{3/2}) \)
10. Which integral below gives the length of the parametric curve?

\[ c(t) = (2e^t - 2t, 8e^{t/2}), \quad t \in [0, 3] \]

(a) \( \int_{0}^{3} (2e^t + 2)^2 \ dt \)

(b) \( \int_{0}^{3} (2e^t + 2) \ dt \)

(c) \( \int_{0}^{3} (e^t + 1) \ dt \)

(d) \( \int_{0}^{3} (4e^t + 4) \ dt \)
11. Find the TL to the PE,

\[ x = te^t, \quad y = e^{2t} \]

at the point \((e, e^2)\).

(a) \( y = ex - e \)
(b) \( y = ex \)
(c) \( y = e^x + e \)
(d) \( y = x + e \)
12. At which angle $\theta$ does the polar curve intersect the origin?

$$r = 4 \cos(3\theta)$$

(a) $\pi/4$.
(b) $\pi/6$.
(c) $4\pi$.
(d) $6\pi$. 
13. Find the area inside the $r_1$ curve, outside the $r_2$ curve in the 1st and 4th quadrants.

\[ r_1^2 = 9 \cos(2\theta), \quad r_2 = \sqrt{6} \cos(\theta) \]

(a) \[ 2 \left( \frac{1}{2} \int_{0}^{\pi/4} r_1^2 - r_2^2 \, d\theta \right) \]

(b) \[ 2 \left( \frac{1}{2} \int_{0}^{\pi/4} r_2^2 - r_1^2 \, d\theta \right) \]

(c) \[ 2 \left( \frac{1}{2} \int_{0}^{\pi/6} r_1^2 - r_2^2 \, d\theta \right) \]

(d) \[ \frac{1}{2} \int_{0}^{\pi/2} r_2^2 - r_1^2 \, d\theta \]
Given flat and polar curves of \( r = \cos \theta + \sin 2\theta \),

let \( \int_{a}^{b} \frac{1}{2}(\cos \theta + \sin 2\theta)^2 \, d\theta \) be the area of the shaded region.

Find \( a \) and \( b \).

A. \( a = \frac{\pi}{2}, \ b = \frac{7\pi}{6} \)

B. \( a = \frac{3\pi}{2}, \ b = \frac{11\pi}{6} \)

C. \( a = \frac{11\pi}{6}, \ b = \frac{\pi}{2} \)

D. \( a = \frac{7\pi}{6}, \ b = \frac{3\pi}{2} \)
Volume:

15. Find the volume of the solid obtained by rotating the region bounded by
   \( y = x^3, \ y = 0, \ x = 1 \)
   about the \( y = -1 \).

(a) \( \int_{0}^{1} \pi [1^2 - (\sqrt[3]{y})^2] \ dy \)

(b) \( \int_{0}^{1} \pi [(\sqrt[3]{y})^2 - 1^2] \ dy \)

(c) \( \int_{0}^{1} \pi [1^2 - (x^3)^2] \ dx \)

(d) \( \int_{0}^{1} \pi [(x^3 + 1)^2 - 1^2] \ dx \)
16. Find the volume of the solid obtained by rotating the region bounded by 
\( y = \sin x, \ y = 0, 0 \leq x \leq \frac{\pi}{2}. \)

a. About the line \( y = 2 \)         b. About the line \( x = -1. \)

c. rotate the region \( y = \sin x, \ y = 0, 0 \leq x \leq \pi. \)
about the line \( x = -1 \)
17. Find the volume of the solid generated by rotating the region bounded by

\[ y = \sqrt{x - 1}, \ y = 0, \ x = 5 \]

and revolved around the line \( y = 3 \).

(a) \( \int_{1}^{5} \pi \left[ (3 - \sqrt{x - 1})^2 - 3^2 \right] \ dx \)

(b) \( \int_{1}^{5} \pi \left[ 3^2 - (\sqrt{x - 1})^2 \right] \ dx \)

(c) \( \int_{1}^{5} \pi \left[ (\sqrt{x - 1})^2 - 3^2 \right] \ dx \)

(d) \( \int_{1}^{5} \pi \left[ 3^2 - (3 - \sqrt{x - 1})^2 \right] \ dx \)
18. Find the volume of the solid if the base of
the solid is the region between the curve \( y =\)
\(2 \sin x\) and the \(x\)—axis on the interval \([0, \pi]\)
and the cross-sections perpendicular to the
\(x\)—axis are squares with bases running from
the \(x\)—axis to the curve.

ex. Find the volume of the solid if the base of
the solid is the region between the curves \( y =\)
\(e^x, y = e^{-x}\) and \(x = 1\) and the cross-sections
perpendicular to the \(x\)—axis are squares.
19. Find the volume of the solid if the base of the solid is the region between the curve $y = 2\sin x$ and the $x$—axis on the interval $[0, \pi]$ and the cross-sections perpendicular to the $x$—axis are semi-circles with bases running from the $x$—axis to the curve.

(a) $\frac{1}{2}\pi$

(b) $\frac{1}{2}\pi^2$

(c) $\frac{1}{4}\pi$

(d) $\frac{1}{4}\pi^2$
20. Find the volume of the solid obtained by rotating the region bounded by 
\[ y = e^x, \quad y = e^{-x}, \quad x = 1 \]
about the \( y \)-axis.

(a) \( \frac{2}{e} \)

(b) \( \frac{4}{e} \)

(c) \( \frac{4\pi}{e} \)

(d) \( \frac{2\pi}{e} \)
21. Find the volume of the solid obtained by rotating the region bounded by
\[ y = \ln x, \ y = 0, \ x = 2 \]
about the \( y \)-axis.

(a) \( 4 \ln 2 - \frac{3}{2} \)

(b) \( \pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(c) \( 2\pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(d) \( 4 \ln 2 - 2 \)
22. Consider the region bounded by the curves $y = -x^2 + x$ and $y = 0$. Using the Shell Method to find the volume of revolution generated by rotating this region about the line $x = 5$. Which integral below is the correct set up?

(a) \[ V = \int_0^1 2\pi(y) \left( \sqrt{\frac{1}{4} - y + \frac{1}{2}} \right) \, dy \]

(b) \[ V = \int_0^1 2\pi(x)(-x^2 + x) \, dx \]

(c) \[ V = \int_0^5 2\pi(5 - x)(-x^2 + x) \, dx \]

(d) \[ V = \int_0^1 2\pi(5 - x)(-x^2 + x) \, dx \]
23. Consider the region bounded by the curves \( y = -x^2 + x \) and \( y = 0 \). Using the Washer Method to find the volume of revolution generated by rotating this region about the line \( y = 3 \). Which choice below gives correct \( OR \) and \( IR \)?

(a) \( OR = 3, \ IR = 3 + (-x^2 + x) \)

(b) \( OR = 3, \ IR = (-x^2 + x) - 3 \)

(c) \( OR = 3, \ IR = 3 - (-x^2 + x) \)
Convergence tests:

24. Consider the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{3 + 2^n} \).

Let \( R_2 \) be the error made in estimating the sum of the series after summing the first 2 terms.

According to the Alternating Series error estimation theorem,

\[ |R_2| \leq \ldots \]

(a) \(-1/11\)
(b) \(1/11\)
(c) \(-1/7\)
(d) \(1/7\)
(e) \(1/19\)
25. Choose the letter of the column whose rows give the statement’s truth value.

<table>
<thead>
<tr>
<th>Summation Function</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \frac{2 + (-1)^n}{10^n}$ conv by DCT</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$\sum \frac{2 + (-1)^n}{n}$ conv by DCT</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \frac{n^3}{2^n(n + 1)}$ conv abs. by LCT</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
26. Determine if the series converge. Choose the letter of the column in the following table whose rows give each statement’s truth value.

<table>
<thead>
<tr>
<th>Series</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \sin\left(\frac{n\pi}{2}\right)$ (cond. conv.)</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \left(\frac{\sin\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)}\right)$ (conv.)</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \frac{\cos n}{n^2}$ (abs. conv)</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \frac{1}{2}$ (conv.)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>$\sum \frac{e^n}{n!}$ (conv.)</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$\sum \sqrt{71}$ (conv.)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
27. Use Direct Comparison test to determine if

\[ \sum_{3}^{\infty} \frac{10 - \cos n}{2^n} \]

is convergent?

(a) convergent
(b) divergent
(c) Can’t use comparison tests
28. **IBP:** Evaluate \( \int \ln x \, dx, \int x^2 \cos x \, dx, \int \arctan x \, dx, \int e^{2x} \sin(3x) \, dx, \int x^3 e^{x^2} \, dx \)

29. **Trig Integrals:** \( \int \sin^3 x \cos^2 x \, dx, \int \cos^2(2x) \, dx, \int \sec^4 x \tan^4 x \, dx, \int \tan^3 x \sec x \, dx, \int \sec x \, dx \)

30. **Trig-sub:** \( \int \frac{1}{\sqrt{1 - x^2}} \, dx, \int \frac{1}{x^2 \sqrt{x^2 + 4}} \, dx \)

\( \int \frac{x}{\sqrt{3 - 2x - x^2}} \, dx, \int \frac{x}{\sqrt{-x^2 + 2x}} \, dx. \)
31. Use an appropriate trig-sub to transform the integral

\[ \int \frac{x^2}{\sqrt{9 - 25x^2}} \, dx \]

into a trig integral for some constant \( k \).

(a) \( I = k \int \sin^2 \theta \, d\theta \)

(b) \( I = k \int \cos^2 \theta \, d\theta \)

(c) \( I = k \int \sin 2\theta \, d\theta \)

(d) \( I = k \int \cos 2\theta \, d\theta \)

(e) \( I = k \int \sin \theta \cos^2 \theta \, d\theta \)
Partial Fractions

32. Split \( \int \frac{x + 4}{x^2 + 2x + 5} \, dx \) into two integrals which can be easily evaluated using the 2-step process (\( u \)-sub and arctangent formula.)

(a) \( \int \frac{x}{x^2 + 2x + 5} \, dx - \int \frac{4}{(x + 1)^2 + 2^2} \, dx \)

(b) \( \int \frac{x + 1}{x^2 + 2x + 5} \, dx + \int \frac{3}{(x + 1)^2 + 2^2} \, dx \)

(c) \( \int \frac{x + 2}{x^2 + 2x + 5} \, dx - \int \frac{2}{(x + 1)^2 + 2^2} \, dx \)

(d) \( \int \frac{x - 1}{x^2 + 2x + 5} \, dx + \int \frac{5}{(x + 1)^2 + 2^2} \, dx \)

(e) \( \int \frac{x - 2}{x^2 + 2x + 5} \, dx + \int \frac{6}{(x + 1)^2 + 2^2} \, dx \)
33. \[ \int \frac{6x - 2}{(x - 3)(x + 2)} \, dx \]