This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:

You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/
1. Investigate the convergence of the series \( \sum_{n=1}^{\infty} \left( \frac{2n+3}{3n+2} \right)^n \).

2. Suppose that \( \sum_{n=0}^{\infty} c_n(x+1)^n \) converges for \( x = 1/2 \) and diverges for \( x = -4 \). Consider:

\[
A = \sum_{n=0}^{\infty} c_n (-5)^n \quad B = \sum_{n=0}^{\infty} c_n \quad C = \sum_{n=0}^{\infty} c_n 2^n
\]

What can be said about the convergence of series \( A \), \( B \), and \( C \)?

3. Investigate the convergence of \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{4n} \).

4. Use the fact that \( 4 \arctan(1) = \pi \), and \( \frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \) to find a series representation for the number \( \pi \). [Hint: integrate]

5. Express \( \int_0^x \frac{e^t - 1 - t}{t^2} \, dt \) as a power series. Find the IOC.

6. Evaluate \( \int \frac{1}{\sqrt{3 - 2x - x^2}} \, dx \).

7. Find the sum of the series \( S = \sqrt{2} - \frac{\sqrt{2} \pi^2}{4^2 \cdot 2!} + \frac{\sqrt{2} \pi^4}{4^4 \cdot 4!} - \frac{\sqrt{2} \pi^6}{4^6 \cdot 6!} + \cdots \)

8. Evaluate the definite integral \( \int_0^1 \sqrt{4 - x^2} \, dx \).

9. Set up the partial fraction decomposition for \( \frac{1}{x^4 - 9x^2} \).
10. Complete the partial fraction decomposition from the previous problem and use it to evaluate \[
\int \frac{1}{x^4 - 9x^2} \, dx.
\]

11. Evaluate \[
\int x^2 \sqrt{x^2 - 4} \, dx.
\]

12. Which of the following integrals converge?

I. \[
\int_2^\infty \frac{1}{x^{6/3}} \, dx
\]

II. \[
\int_1^\infty \frac{e^t}{1 + e^{4t}} \, dt
\]

III. \[
\int_2^\infty \frac{1}{\ln(s)} \, ds
\]

13. Find a closed form for the \(N^{th}\) partial sum of \[
\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right)
\]

14. Sketch the polar curve \(r = 1 + \cos(\theta)\), and set up an integral to find the area above the horizontal axis.

15. Given \(r = 1 - 2 \cos(\theta)\) set up an integral for the area of the inner loop.

16. Set up an integral for the area inside \(r_2\) and outside \(r_1\) where \(r_1 = \sqrt{3} \sin(\theta)\) and \(r_2 = \cos(\theta)\).

17. Given \(x(t) = e^t + 5t\) and \(y(t) = 100\), find the arclength from \(t = 0\) to \(t = 5\).

18. At what points (if any) does the parametric curve have horizontal or vertical tangent lines?

\[
x(t) = \cos^2(t) + \cos(t) \quad y(t) = \sin(t) \cos(t) + \sin(t)
\]

19. Consider the solid obtained by rotating the region bounded by \(y = \ln(x), x = e,\) and \(y = 0\) about the \(x\)-axis. Set up two integrals for the volume of this solid. One using the disk/washer method, the other using cylindrical shells.
Power Series

1. Evaluate \( \int \frac{t}{1 - t^7} \, dt \) as an infinite series.

2. Suppose \( \sum c_n x^n \) converges at \( x = 6 \). Which below MUST be convergent?

   P: \( \sum c_n (-4)^n \), \( \sum c_n (-6)^n \)

   (a) P only
   (b) Q only
   (c) Both
   (d) Neither
3. Suppose \( \sum c_n x^n \) converges at \( x = 6 \). Which below is NOT possible?

\[
\text{P: } \sum c_n (-4)^n, \text{ diverges} \\
\text{Q: } \sum c_n (-6)^n, \text{ converges}
\]

(a) P only  
(b) Q only  
(c) Both  
(d) Neither
4. Find a power series for $e^{2x}$, centered at 6.

(a) \[ \sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!} \]

(b) \[ \sum_{n=0}^{\infty} e^6 \frac{(x - 6)^n}{n!} \]

(c) \[ e^{12} \sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!} \]

(d) \[ e^{12} \sum_{n=0}^{\infty} \frac{2^n (x - 6)^n}{n!} \]
5. Find a power series for $f(x)$, centered at 0.

$$f(x) = \frac{x}{(1 + x)^2}$$
6. Find the **sum** of the series

\[
\sum_{n=2}^{\infty} \left( \frac{-2^{2n}}{(-9)^{n-1}} + \frac{(-1)^n}{(2n)!} \right)
\]

(a) \( \frac{16}{13} + \cos 1 \)

(b) \( -\frac{16}{11} - \cos 1 \)

(c) \( \frac{16}{11} + e \)

(d) \( -\frac{16}{13} - \sin 1 \)

(e) \( \frac{11}{9} + \cos 1 \)
7. Find the value of $t$ for which the series $\sum_{n=0}^{\infty} \frac{(-1)^n t^{n+1}}{3^{2n}(2n + 1)}$ converges, and find the function it converges to for these values of $t$.

(ans. $f(t) = t \arctan\left(\frac{t}{3}\right), t \in [-3.3]$)
8. (TRUE/FALSE)

Let

\[ f(0) = 4, \quad f'(0) = 3, \]
\[ f''(0) = 3, \quad f'''(0) = 2. \]

The first 4 terms of the Maclaurin Series of \( f \) is

\[ f(x) = 4 + 3x + 3x^2 + 2x^3 \]

(a) TRUE

(b) FALSE
Parametric and Polar:

9. Find the length of the path over the given interval.

\[ c(t) = (t^3 + 8, t^2 + 3), \quad 0 \leq t \leq 1 \]

(a) \((100 + \frac{4}{9})^{3/2} - (25 + \frac{4}{9})^{3/2}\)

(b) \((100 + \frac{9}{4})^{3/2} - (25 + \frac{9}{4})^{3/2}\)

(c) \((100 + \frac{4}{9})^{3/2} + (25 + \frac{4}{9})^{3/2}\)

(d) \(\frac{1}{27}(13^{3/2} - 4^{3/2})\)
10. Which integral below gives the length of the parametric curve?

\[ c(t) = (2e^t - 2t, 8e^{t/2}), \quad t \in [0, 3] \]

(a) \[ \int_{0}^{3} (2e^t + 2)^2 \, dt \]

(b) \[ \int_{0}^{3} (2e^t + 2) \, dt \]

(c) \[ \int_{0}^{3} (e^t + 1) \, dt \]

(d) \[ \int_{0}^{3} (4e^t + 4) \, dt \]
11. Find the TL to the PE,

\[ x = te^t, \quad y = e^{2t} \]

at the point \((e, e^2)\).

(a) \(y = ex - e\)

(b) \(y = ex\)

(c) \(y = e^x + e\)

(d) \(y = x + e\)
12. At which angle $\theta$ does the polar curve intersect the origin?

$r = 4 \cos(3\theta)$

(a) $\pi/4$.

(b) $\pi/6$.

(c) $4\pi$.

(d) $6\pi$. 
13. Find the area inside the $r_1$ curve, outside the $r_2$ curve in the 1st and 4th quadrants.

\[ r_1^2 = 9 \cos(2\theta), \quad r_2 = \sqrt{6} \cos(\theta) \]

(a) \(2 \left( \frac{1}{2} \int_0^{\pi/4} r_1^2 - r_2^2 \, d\theta \right)\)

(b) \(2 \left( \frac{1}{2} \int_0^{\pi/4} r_2^2 - r_1^2 \, d\theta \right)\)

(c) \(2 \left( \frac{1}{2} \int_0^{\pi/6} r_1^2 - r_2^2 \, d\theta \right)\)

(d) \(\frac{1}{2} \int_0^{\pi/2} r_2^2 - r_1^2 \, d\theta\)
Given flat and polar curves of \( r = \cos \theta + \sin 2\theta \),

let \( \int_{a}^{b} \frac{1}{2}(\cos \theta + \sin 2\theta)^2 d\theta \) be the area of the shaded region.

Find \( a \) and \( b \).

\[
\begin{align*}
A. \quad & a = \frac{\pi}{2}, \quad b = \frac{7\pi}{6} \\
B. \quad & a = \frac{3\pi}{2}, \quad b = \frac{11\pi}{6} \\
C. \quad & a = \frac{11\pi}{6}, \quad b = \frac{\pi}{2} \\
D. \quad & a = \frac{7\pi}{6}, \quad b = \frac{3\pi}{2}
\end{align*}
\]
Volume:
15. Find the volume of the solid obtained by rotating the region bounded by
\( y = x^3, \ y = 0, \ x = 1 \)
about the \( y = -1 \).

(a) \( \int_{0}^{1} \pi [1^2 - (\sqrt[3]{y})^2] \, dy \)

(b) \( \int_{0}^{1} \pi [(\sqrt[3]{y})^2 - 1^2] \, dy \)

(c) \( \int_{0}^{1} \pi [1^2 - (x^3)^2] \, dx \)

(d) \( \int_{0}^{1} \pi [(x^3 + 1)^2 - 1^2] \, dx \)
16. Find the volume of the solid obtained by rotating the region bounded by 
\( y = \sin x, \ y = 0, \ 0 \leq x \leq \frac{\pi}{2} \).

a. About the line \( y = 2 \)  

b. About the line \( x = -1 \).

c. Rotate the region \( y = \sin x, \ y = 0, \ 0 \leq x \leq \pi \)  
about the line \( x = -1 \).
17. Find the volume of the solid generated by rotating the region bounded by

\[ y = \sqrt{x - 1}, \ y = 0, \ x = 5 \]

and revolved around the line \( y = 3 \).

(a) \[ \int_{1}^{5} \pi \left[ (3 - \sqrt{x-1})^2 - 3^2 \right] \ dx \]

(b) \[ \int_{1}^{5} \pi \left[ 3^2 - (\sqrt{x-1})^2 \right] \ dx \]

(c) \[ \int_{1}^{5} \pi \left[ (\sqrt{x-1})^2 - 3^2 \right] \ dx \]

(d) \[ \int_{1}^{5} \pi \left[ 3^2 - (3 - \sqrt{x-1})^2 \right] \ dx \]
18. Find the volume of the solid if the base of the solid is the region between the curve $y = 2 \sin x$ and the $x$–axis on the interval $[0, \pi]$ and the cross-sections perpendicular to the $x$–axis are squares with bases running from the $x$–axis to the curve.

ex. Find the volume of the solid if the base of the solid is the region between the curves $y = e^x$, $y = e^{-x}$ and $x = 1$ and the cross-sections perpendicular to the $x$–axis are squares.
19. Find the volume of the solid if the base of the solid is the region between the curve 
\( y = 2 \sin x \) and the \( x \)-axis on the interval \([0, \pi]\) and the cross-sections perpendicular to the \( x \)-axis are semi-circles with bases running from the \( x \)-axis to the curve.

(a) \( \frac{1}{2}\pi \)

(b) \( \frac{1}{2}\pi^2 \)

(c) \( \frac{1}{4}\pi \)

(d) \( \frac{1}{4}\pi^2 \)
20. Find the volume of the solid obtained by rotating the region bounded by

\[ y = e^x, \; y = e^{-x}, \; x = 1 \]

about the \( y \)-axis.

(a) \( \frac{2}{e} \)

(b) \( \frac{4}{e} \)

(c) \( \frac{4\pi}{e} \)

(d) \( \frac{2\pi}{e} \)
21. Find the volume of the solid obtained by rotating the region bounded by 

\[ y = \ln x, \ y = 0, \ x = 2 \]

about the \( y \)-axis.

(a) \( 4 \ln 2 - \frac{3}{2} \)

(b) \( \pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(c) \( 2\pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(d) \( 4 \ln 2 - 2 \)
22. Consider the region bounded by the curves \( y = -x^2 + x \) and \( y = 0 \). Using the Shell Method to find the volume of revolution generated by rotating this region about the line \( x = 5 \). Which integral below is the correct set up?

(a) \( V = \int_0^1 2\pi (y) \left( \sqrt{\frac{1}{4} - y} + \frac{1}{2} \right) \, dy \)

(b) \( V = \int_0^1 2\pi (x)(-x^2 + x) \, dx \)

(c) \( V = \int_0^5 2\pi (5 - x)(-x^2 + x) \, dx \)

(d) \( V = \int_0^1 2\pi (5 - x)(-x^2 + x) \, dx \)
23. Consider the region bounded by the curves \( y = -x^2 + x \) and \( y = 0 \). Using the \textbf{Washer Method} to find the volume of revolution generated by rotating this region about the line \( y = 3 \). Which choice below gives correct \( OR \) and \( IR \)?

(a) \( OR = 3, \ IR = 3 + (-x^2 + x) \)

(b) \( OR = 3, \ IR = (-x^2 + x) - 3 \)

(c) \( OR = 3, \ IR = 3 - (-x^2 + x) \)
Convergence tests:

24. Consider the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{3 + 2^n} \).

Let \( R_2 \) be the error made in estimating the sum of the series after summing the first 2 terms.

According to the Alternating Series error estimation theorem,

\[ |R_2| \leq \text{____}. \]

(a) \(-1/11\)  
(b) \(1/11\)  
(c) \(-1/7\)  
(d) \(1/7\)  
(e) \(1/19\)
25. Choose the letter of the column whose rows give the statement’s truth value.

<table>
<thead>
<tr>
<th>Expression</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \frac{2 + (-1)^n}{10^n}$ conv by DCT</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$\sum \frac{2 + (-1)^n}{n}$ conv by DCT</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \frac{n^3}{2^n(n+1)}$ conv abs. by LCT</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
26. Determine if the series converge. Choose the letter of the column in the following table whose rows give each statement’s truth value.

<table>
<thead>
<tr>
<th>Series</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum \sin\left(\frac{n\pi}{2}\right) ]</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>[ \sum \left( \frac{\sin\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} \right) ]</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>[ \sum \frac{\cos n}{n^2} ] abs. conv.</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>[ \sum \frac{1}{2} ] conv.</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>[ \sum e^n ] conv.</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>[ \sum \sqrt{71} ] conv.</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
27. Use Direct Comparison test to determine if

\[ \sum_{3}^{\infty} \frac{10 - \cos n}{2^n} \]

is convergent?

(a) convergent
(b) divergent
(c) Can’t use comparison tests
28. **IBP:** Evaluate \[
\int \ln x \, dx, \quad \int x^2 \cos x \, dx,
\]
\[
\int \arctan x \, dx, \quad \int e^{2x} \sin(3x) \, dx, \quad \int x^3 e^{x^2} \, dx
\]

29. **Trig Integrals:** \[
\int \sin^3 x \cos^2 x \, dx, \quad \int \cos^2(2x) \, dx,
\]
\[
\int \sec^4 x \tan^4 x \, dx, \quad \int \tan^3 x \sec x \, dx, \quad \int \sec x \, dx
\]

30. **Trig-sub:** \[
\int \frac{1}{\sqrt{1-x^2}} \, dx, \quad \int \frac{1}{x^2 \sqrt{x^2 + 4}} \, dx
\]
\[
\int \frac{x}{\sqrt{3-2x-x^2}} \, dx, \quad \int \frac{x}{\sqrt{-x^2 + 2x}} \, dx.
\]
31. Use an appropriate trig-sub to transform the integral

\[ \int \frac{x^2}{\sqrt{9 - 25x^2}} \, dx \]

into a trig integral for some constant \( k \).

(a) \( I = k \int \sin^2 \theta \, d\theta \)

(b) \( I = k \int \cos^2 \theta \, d\theta \)

(c) \( I = k \int \sin 2\theta \, d\theta \)

(d) \( I = k \int \cos 2\theta \, d\theta \)

(e) \( I = k \int \sin \theta \cos^2 \theta \, d\theta \)
Partial Fractions

32. Split \( \int \frac{x + 4}{x^2 + 2x + 5} \, dx \) into two integrals which can be easily evaluated using the 2-step process (\( u \)-sub and arctangent formula.)

(a) \( \int \frac{x}{x^2 + 2x + 5} \, dx - \int \frac{4}{(x + 1)^2 + 2^2} \, dx \)

(b) \( \int \frac{x + 1}{x^2 + 2x + 5} \, dx + \int \frac{3}{(x + 1)^2 + 2^2} \, dx \)

(c) \( \int \frac{x + 2}{x^2 + 2x + 5} \, dx - \int \frac{2}{(x + 1)^2 + 2^2} \, dx \)

(d) \( \int \frac{x - 1}{x^2 + 2x + 5} \, dx + \int \frac{5}{(x + 1)^2 + 2^2} \, dx \)

(e) \( \int \frac{x - 2}{x^2 + 2x + 5} \, dx + \int \frac{6}{(x + 1)^2 + 2^2} \, dx \)
33. \[ \int \frac{6x - 2}{(x - 3)(x + 2)} \, dx \]