This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:

You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/
1. Investigate the convergence of the series \( \sum_{n=1}^{\infty} \left( \frac{2n+3}{3n+2} \right)^n \).

2. Suppose that \( \sum_{n=0}^{\infty} c_n(x+1)^n \) converges for \( x = 1/2 \) and diverges for \( x = -4 \). Consider:

\[
A = \sum_{n=0}^{\infty} c_n(-5)^n \quad B = \sum_{n=0}^{\infty} c_n \quad C = \sum_{n=0}^{\infty} c_n 2^n
\]

What can be said about the convergence of series \( A \), \( B \), and \( C \)?

3. Investigate the convergence of \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{4n} \).

4. Use the fact that \( 4 \arctan(1) = \pi \), and \( \frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \) to find a series representation for the number \( \pi \). [Hint: integrate]

5. Express \( \int_0^{x} \frac{e^t - 1 - t}{t^2} \, dt \) as a power series. Find the IOC.

6. Evaluate \( \int \frac{1}{\sqrt{3 - 2x - x^2}} \, dx \)

7. Find the sum of the series \( S = \sqrt{2} - \frac{\sqrt{2} \pi^2}{4^2 \cdot 2!} + \frac{\sqrt{2} \pi^4}{4^4 \cdot 4!} - \frac{\sqrt{2} \pi^6}{4^6 \cdot 6!} + \cdots \)

8. Evaluate the definite integral \( \int_0^{1} \sqrt{4 - x^2} \, dx \).

9. Set up the partial fraction decomposition for \( \frac{1}{x^4 - 9x^2} \).
10. Complete the partial fraction decomposition from the previous problem and use it to evaluate
\[ \int \frac{1}{x^4 - 9x^2} \, dx. \]

11. Evaluate \[ \int x^2 \sqrt{x^2 - 4} \, dx \]

12. Which of the following integrals converge?

I. \[ \int_2^\infty \frac{1}{x^{\pi/3}} \, dx \]

II. \[ \int_1^\infty \frac{e^t}{1 + e^{4t}} \, dt \]

III. \[ \int_2^\infty \frac{1}{\ln(s)} \, ds \]

13. Find a closed form for the \( N^{th} \) partial sum of \[ \sum_{n=2}^{\infty} \ln\left( \frac{n}{n+1} \right) \]

14. Sketch the polar curve \( r = 1 + \cos(\theta) \), and set up an integral to find the area above the horizontal axis.

15. Given \( r = 1 - 2 \cos(\theta) \) set up an integral for the area of the inner loop.

16. Set up an integral for the area inside \( r_2 \) and outside \( r_1 \) where \( r_1 = \sqrt{3} \sin(\theta) \) and \( r_2 = \cos(\theta) \).

17. Given \( x(t) = e^t + 5t \) and \( y(t) = 100 \), find the arclength from \( t = 0 \) to \( t = 5 \).

18. At what points (if any) does the parametric curve have horizontal or vertical tangent lines?

\[ x(t) = \cos^2(t) + \cos(t) \quad y(t) = \sin(t) \cos(t) + \sin(t) \]

19. Consider the solid obtained by rotating the region bounded by \( y = \ln(x) \), \( x = e \), and \( y = 0 \) about the \( x \)-axis. Set up two integrals for the volume of this solid. One using the disk/washer method, the other using cylindrical shells.
Fall 2018 Final Exam is roughly broken down as follows:

Integration unit (3) (+1)
Convergence tests (5)
Power Series (3)
Parametric (3)
Polar (3)
Volume (5)

Final contains: All multiple choices. 20 questions worth 3 points/ea and 2 extra credit questions worth 1.5 points/ea.

'Maclaurin Series' sheet is provided on the final.
Power Series

1. Evaluate \( \int \frac{t}{1-t^7} \, dt \) as an infinite series.

2. Suppose \( \sum c_n x^n \) converges at \( x = 6 \). Which below MUST be convergent?

   P: \( \sum c_n (-4)^n \), \( Q: \sum c_n (-6)^n \)

   (a) P only
   (b) Q only
   (c) Both
   (d) Neither
3. Suppose $\sum c_n x^n$ converges at $x = 6$. Which below is NOT possible?

P: $\sum c_n (-4)^n$, diverges

Q: $\sum c_n (-6)^n$ converges

(a) P only

(b) Q only

(c) Both

(d) Neither
4. Find a power series for $e^{2x}$, centered at 6.

(a) $\sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!}$

(b) $\sum_{n=0}^{\infty} e^6 \frac{(x - 6)^n}{n!}$

(c) $e^{12} \sum_{n=0}^{\infty} \frac{(x - 6)^n}{n!}$

(d) $e^{12} \sum_{n=0}^{\infty} \frac{2^n(x - 6)^n}{n!}$
5. Find a power series for $f(x)$, centered at 0.

$$f(x) = \frac{x}{(1 + x)^2}$$
6. Find the **sum** of the series

\[
\sum_{n=2}^{\infty} \left( \frac{-2^{2n}}{(-9)^{n-1}} + \frac{(-1)^n}{(2n)!} \right)
\]

(a) \(\frac{16}{13} + \cos 1\)

(b) \(-\frac{16}{11} - \cos 1\)

(c) \(\frac{16}{11} + e\)

(d) \(-\frac{16}{13} - \sin 1\)

(e) \(\frac{11}{9} + \cos 1\)
7. Find the value of $t$ for which the series \[ \sum_{n=0}^{\infty} \frac{(-1)^n t^{n+1}}{3^{2n}(2n + 1)} \]
converges, and find the function it converges to for these values of $t$.
(ans. $f(t) = t \arctan \left( \frac{t}{3} \right), t \in [-3.3]$)
8. (TRUE/FALSE)

Let

\[ f(0) = 4, \quad f'(0) = 3, \]
\[ f''(0) = 3, \quad f'''(0) = 2. \]

The first 4 terms of the Maclaurin Series of \( f \) is

\[ f(x) = 4 + 3x + 3x^2 + 2x^3 \]

(a) TRUE

(b) FALSE
Parametric and Polar:

9. Find the length of the path over the given interval.

\[ c(t) = (t^3 + 8, t^2 + 3), \quad 0 \leq t \leq 1 \]

(a) \((100 + \frac{4}{9})^{3/2} - (25 + \frac{4}{9})^{3/2}\)

(b) \((100 + \frac{9}{4})^{3/2} - (25 + \frac{9}{4})^{3/2}\)

(c) \((100 + \frac{4}{9})^{3/2} + (25 + \frac{4}{9})^{3/2}\)

(d) \(\frac{1}{27}(13^{3/2} - 4^{3/2})\)
10. Which integral below gives the length of the parametric curve?

\[ c(t) = (2e^t - 2t, 8e^{t/2}), \quad t \in [0, 3] \]

(a) \[ \int_0^3 (2e^t + 2)^2 \, dt \]
(b) \[ \int_0^3 (2e^t + 2) \, dt \]
(c) \[ \int_0^3 (e^t + 1) \, dt \]
(d) \[ \int_0^3 (4e^t + 4) \, dt \]
11. Find the TL to the PE,

\[ x = te^t, \quad y = e^{2t} \]

at the point \((e, e^2)\).

(a) \( y = ex - e \)
(b) \( y = ex \)
(c) \( y = e^x + e \)
(d) \( y = x + e \)
12. At which angle $\theta$ does the polar curve intersect the origin?

$$r = 4 \cos(3\theta)$$

(a) $\pi/4$.

(b) $\pi/6$.

(c) $4\pi$.

(d) $6\pi$. 

13. Find the area inside the \( r_1 \) curve, outside the \( r_2 \) curve in the 1st and 4th quadrants.

\[ r_1^2 = 9 \cos(2\theta), \quad r_2 = \sqrt{6} \cos(\theta) \]

(a) \( 2 \left( \frac{1}{2} \int_0^{\pi/4} r_1^2 - r_2^2 \, d\theta \right) \)

(b) \( 2 \left( \frac{1}{2} \int_0^{\pi/4} r_2^2 - r_1^2 \, d\theta \right) \)

(c) \( 2 \left( \frac{1}{2} \int_0^{\pi/6} r_1^2 - r_2^2 \, d\theta \right) \)

(d) \( \frac{1}{2} \int_0^{\pi/2} r_2^2 - r_1^2 \, d\theta \)
Given flat and polar curves of $r = \cos \theta + \sin 2\theta$,

let \( \int_{a}^{b} \frac{1}{2}(\cos \theta + \sin 2\theta)^2 \, d\theta \) be the area of the shaded region.

Find \( a \) and \( b \).

\[ r = \cos \theta + \sin 2\theta \]

A. \( a = \frac{\pi}{2}, \ b = \frac{7\pi}{6} \)

B. \( a = \frac{3\pi}{2}, \ b = \frac{11\pi}{6} \)

C. \( a = \frac{11\pi}{6}, \ b = \frac{\pi}{2} \)

D. \( a = \frac{7\pi}{6}, \ b = \frac{3\pi}{2} \)
Volume:

15. Find the volume of the solid obtained by rotating the region bounded by 
   \[ y = x^3, \ y = 0, \ x = 1 \]
   about the \( y = -1 \).

(a) \[ \int_0^1 \pi [1^2 - (\sqrt[3]{y})^2] \, dy \]

(b) \[ \int_0^1 \pi [(\sqrt[3]{y})^2 - 1^2] \, dy \]

(c) \[ \int_0^1 \pi [1^2 - (x^3)^2] \, dx \]

(d) \[ \int_0^1 \pi [(x^3 + 1)^2 - 1^2] \, dx \]
16. Find the volume of the solid obtained by rotating the region bounded by
\( y = \sin x, \quad y = 0, \quad 0 \leq x \leq \frac{\pi}{2} \).

a. About the line \( y = 2 \)  

b. About the line \( x = -1 \).

c. Rotate the region \( y = \sin x, \quad y = 0, \quad 0 \leq x \leq \pi \) about the line \( x = -1 \)
17. Find the volume of the solid generated by rotating the region bounded by

\[ y = \sqrt{x - 1}, \ y = 0, \ x = 5 \]

and revolved around the line \( y = 3 \).

(a) \( \int_{1}^{5} \pi \left[ (3 - \sqrt{x - 1})^2 - 3^2 \right] \ dx \)

(b) \( \int_{1}^{5} \pi \left[ 3^2 - (\sqrt{x - 1})^2 \right] \ dx \)

(c) \( \int_{1}^{5} \pi \left[ (\sqrt{x - 1})^2 - 3^2 \right] \ dx \)

(d) \( \int_{1}^{5} \pi \left[ 3^2 - (3 - \sqrt{x - 1})^2 \right] \ dx \)
18. Find the volume of the solid if the base of the solid is the region between the curve \( y = 2 \sin x \) and the \( x \)-axis on the interval \([0, \pi]\) and the cross-sections perpendicular to the \( x \)-axis are squares with bases running from the \( x \)-axis to the curve.

ex. Find the volume of the solid if the base of the solid is the region between the curves \( y = e^x, \ y = e^{-x} \) and \( x = 1 \) and the cross-sections perpendicular to the \( x \)-axis are squares.
19. Find the volume of the solid if the base of the solid is the region between the curve $y = 2\sin x$ and the $x$—axis on the interval $[0, \pi]$ and the cross-sections perpendicular to the $x$—axis are semi-circles with bases running from the $x$—axis to the curve.

(a) $\frac{1}{2} \pi$

(b) $\frac{1}{2} \pi^2$

(c) $\frac{1}{4} \pi$

(d) $\frac{1}{4} \pi^2$
20. Find the volume of the solid obtained by rotating the region bounded by 
\[ y = e^x, \quad y = e^{-x}, \quad x = 1 \]
about the \( y \)-axis.

\begin{align*}
(a) & \frac{2}{e} \\
(b) & \frac{4}{e} \\
(c) & \frac{4\pi}{e} \\
(d) & \frac{2\pi}{e}
\end{align*}
21. Find the volume of the solid obtained by rotating the region bounded by
\[ y = \ln x, \quad y = 0, \quad x = 2 \]
about the \( y \)-axis.

(a) \( 4 \ln 2 - \frac{3}{2} \)

(b) \( \pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(c) \( 2\pi \left( 4 \ln 2 - \frac{3}{2} \right) \)

(d) \( 4 \ln 2 - 2 \)
22. Consider the region bounded by the curves \( y = -x^2 + x \) and \( y = 0 \). Using the Shell Method to find the volume of revolution generated by rotating this region about the line \( x = 5 \). Which integral below is the correct set up?

(a) \( V = \int_0^1 2\pi(y) \left( \sqrt{\frac{1}{4} - y + \frac{1}{2}} \right) \, dy \)

(b) \( V = \int_0^1 2\pi(x)(-x^2 + x) \, dx \)

(c) \( V = \int_0^5 2\pi(5 - x)(-x^2 + x) \, dx \)

(d) \( V = \int_0^1 2\pi(5 - x)(-x^2 + x) \, dx \)
23. Consider the region bounded by the curves \( y = -x^2 + x \) and \( y = 0 \). Using the Washer Method to find the volume of revolution generated by rotating this region about the line \( y = 3 \). Which choice below gives correct \( OR \) and \( IR \)?

(a) \( OR = 3, \ IR = 3 + (-x^2 + x) \)

(b) \( OR = 3, \ IR = (-x^2 + x) - 3 \)

(c) \( OR = 3, \ IR = 3 - (-x^2 + x) \)
Convergence tests:

24. Consider the series \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{3 + 2^n}. \]

Let \( R_2 \) be the error made in estimating the sum of the series after summing the first 2 terms.

According to the Alternating Series error estimation theorem,

\[ |R_2| \leq \ldots. \]

(a) \(-1/11\)
(b) \(1/11\)
(c) \(-1/7\)
(d) \(1/7\)
(e) \(1/19\)
25. Choose the letter of the column whose rows give the statement’s truth value.

<table>
<thead>
<tr>
<th>Expression</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \frac{2 + (-1)^n}{10^n}$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$\sum \frac{2 + (-1)^n}{n}$</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>$\sum \frac{n^3}{2^n(n+1)}$</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
26. Determine if the series converge. Choose the letter of the column in the following table whose rows give each statement’s truth value.

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum \sin\left(\frac{n\pi}{2}\right) ) cond. conv.</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>( \sum \left(\frac{\sin\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)}\right) ) conv.</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>( \sum \frac{\cos n}{n^2} ) abs. conv</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>( \sum \frac{1}{2} ) conv.</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>( \sum \frac{e^n}{n!} ) conv.</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>( \sum \sqrt{71} ) conv.</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
27. Use Direct Comparison test to determine if

\[ \sum_{3}^{\infty} \frac{10 - \cos n}{2^n} \]

is convergent?

(a) convergent
(b) divergent
(c) Can’t use comparison tests
28. **IBP:** Evaluate \( \int \ln x \, dx, \int x^2 \cos x \, dx, \int \arctan x \, dx, \int e^{2x} \sin(3x) \, dx, \int x^3 e^{x^2} \, dx \)

29. **Trig Integrals:** \( \int \sin^3 x \cos^2 x \, dx, \int \cos^2(2x) \, dx, \int \sec^4 x \tan^4 x \, dx, \int \tan^3 x \sec x \, dx, \int \sec x \, dx \)

30. **Trig-sub:** \( \int \frac{1}{\sqrt{1 - x^2}} \, dx, \int \frac{1}{x^2 \sqrt{x^2 + 4}} \, dx \)
\( \int \frac{x}{\sqrt{3 - 2x - x^2}} \, dx, \int \frac{x}{\sqrt{-x^2 + 2x}} \, dx. \)
31. Use an appropriate trig-sub to transform the integral
\[ \int \frac{x^2}{\sqrt{9 - 25x^2}} \, dx \]
into a trig integral for some constant \( k \).

(a) \( I = k \int \sin^2 \theta \, d\theta \)
(b) \( I = k \int \cos^2 \theta \, d\theta \)
(c) \( I = k \int \sin 2\theta \, d\theta \)
(d) \( I = k \int \cos 2\theta \, d\theta \)
(e) \( I = k \int \sin \theta \cos^2 \theta \, d\theta \)
Partial Fractions

32. Split \( \int \frac{x + 4}{x^2 + 2x + 5} \, dx \) into two integrals which can be easily evaluated using the 2-step process \((u\text{-sub and arctangent formula.})\)

(a) \( \int \frac{x}{x^2 + 2x + 5} \, dx - \int \frac{4}{(x + 1)^2 + 2^2} \, dx \)

(b) \( \int \frac{x + 1}{x^2 + 2x + 5} \, dx + \int \frac{3}{(x + 1)^2 + 2^2} \, dx \)

(c) \( \int \frac{x + 2}{x^2 + 2x + 5} \, dx - \int \frac{2}{(x + 1)^2 + 2^2} \, dx \)

(d) \( \int \frac{x - 1}{x^2 + 2x + 5} \, dx + \int \frac{5}{(x + 1)^2 + 2^2} \, dx \)

(e) \( \int \frac{x - 2}{x^2 + 2x + 5} \, dx + \int \frac{6}{(x + 1)^2 + 2^2} \, dx \)
33. \[ \int \frac{6x - 2}{(x - 3)(x + 2)} \, dx \]