Important to Remember: If $a$ and $b$ are real numbers,

I. Absolute Value:

$$|a| = \begin{cases} 
-a & a < 0 \\
\quad a & a \geq 0 
\end{cases}$$

Note that: 1) $|a| \geq 0$ 2) $-a = |a|$ 3) $|a| = \left| \frac{a}{b} \right|$ for $b \neq 0$, 4) $|ab| = |a||b|

II. distance between $a$ and $b$ on the number line is $d(a,b) = |b - a|

III. $\sqrt[n]{x^n} = |x|$ if $n$ is even, $\sqrt[n]{x^n} = x$ if $x$ is odd, $\sqrt[n]{x^n} = x$ if the root is defined

III. Special Products and Factoring

If $u$ and $v$ are numbers, variables or expressions:

1) $(u + v)^2 = u^2 + 2uv + v^2$ and $(u - v)^2 = u^2 - 2uv + v^2$
2) $u^2 - v^2 = (u - v)(u + v)$
3) $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$ and $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$

Exercises:

1. Find: a) $(-4)^2$ b) $-4^2$ c) $(-4)^{3/2}$ d) $(-8)^{1/3}$ e) $(-125)^{2/3}$ f) $\sqrt[3]{-4}$

2. If $A = \left\{ \frac{0}{4}, \sqrt{25}, \frac{\pi}{4}, 0.3, \sqrt{-27}, -\frac{2}{3}, \sqrt{-9}, \frac{3}{0}, \sqrt[3]{0.202002000...} \right\}$

Classify the subsets of $A$ which are

(a) Integers
(b) Rational numbers
(c) Irrational numbers
(d) Real numbers
3. Express the following both in interval notation and as an inequality. Be sure to simplify.
   a) $x$ is less than 2 but no less than $-3$  
   b) $x$ is at most 5 units from $-3$

4. Write without absolute value bars and simplify:
   a) $|\pi - 5| - |3 - \pi|$  
   b) $3|12 - 4r| + |r - 1|$ if $r > 3$  
   c) $\frac{2x + |-x^2 - 1|}{x + 1}$

5. If $x = -1$ and $y = 3$ evaluate:
   a) $\frac{|2x - y| - (2x)^2}{|x + y| - |x - y|}$  
   b) $\frac{x^{-1} + y^{-2}}{x^{-4} - y^{-3}}$

6. Rewrite as a piecewise expression without absolute value bars: $\frac{|2x + 5|}{2x + 5}$

7. Evaluate: $356 \cdot (0.24) - 523 \cdot (0.86) + 644 \cdot (0.24) - 477 \cdot (0.86)$

8. Evaluate: $9 \left( -\frac{27}{64} \right)^{-2/3} + 6(-2)^0 \left( \frac{1}{6} \right)^{-1/4}$

9. Simplify the expression:
   a) $(3abc)^{-1} \left( \frac{2a^2b^{-1}}{c^3} \right)^{-2}$  
   b) $\frac{27(-3xy^{-2})^{-3}}{x^2y^4}$  
   c) $\left( \frac{16x^{-10}}{x^6} \right)^{-1/4}$  
   d) $\left( \frac{6x^{-3/2}y^{-1}}{8x^{1/2}y^{-2}} \right)^{-2}$

10. Simplify each radical expression. Be sure to use absolute value as necessary (hint: consider the possible domain of each variable).
    a) $\sqrt[3]{a^4b^{11}c^6}$  
    b) $\sqrt[4]{a^4b^8c^5}$  
    c) $\sqrt[4]{\frac{8a^8}{b^{14}}}$  
    d) $\sqrt[3]{\frac{6v^{-3}}{24v^3}}$

11. Simplify each radical expression and rationalize the denominator:
    a) $\sqrt[5]{\frac{81a^2b^4}{z^5}}$  
    b) $\sqrt[5]{\frac{-4x^{10}y^{12}}{64x^{-5}z^3y^{19}}}$

12. Rationalize the denominator of each expression:
    a) $\frac{1}{3\sqrt{x^2}}$  
    b) $\frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$  
    c) $\frac{\sqrt{10} - \sqrt{8}}{2 + \sqrt{2}}$

13. Simplify each expression if possible:
    a) $\sqrt{80x^3} + \sqrt{125x^2} + x\sqrt{180x}$  
    b) $\sqrt{a^2 + b^2}$

14. Perform the operation: a) $(2x + 3)^3$  
    b) $(m - 3 + n)(m - 3 - n)$
15. Factor completely:
   a) \(4x^3(3-2x)^2 - 8x^2(3-2x)(3x-1)\)
   b) \(x^4 - x^2 - 12\)
   c) \(20 - y - y^2\)
   d) \(\frac{1}{2}x^3 + 2x^2 - 6x\) (factor so there are no fractional coefficients)
   e) \(4x^4 - x^2 - 16x^3 + 4x\)
   f) \(32x^5 - 4x^2\)
   g) \(5x^2 - 26x + 5\)
   h) \((2x - 1)^2 - 9y^6\)
   i) \(9x^2 + 24xy + 16y^2\)
   j) Factor using grouping only: \(6x^2 - x - 15\)

16. Factor the expression with negative exponents: remember to factor out the least power of the common variable/expression
   a) \(15x^{-2}(2x + 1)^{-5} - 10x^{-1}(2x + 1)^{-7}\)

17. Find the domain of the expression and reduce to its lowest terms:
   a) \(\frac{2x^2 - x^3 - 2 + x}{x^2 - 3x - 4}\)
   b) \(\frac{12y^2 - 4y - 5}{6y^2 - y - 2}\)

18. Perform the operation and simplify. Find all excluded values of the variable.
   Note: In the factoring formula \(a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)\) the trinomial will not factor (if \(a\) and \(b\) do not have a common factor).
   a) \(\frac{4x^2 - 4}{x^2 - x + 1} \cdot \frac{x^4 + x}{x^2 + 2x + 1}\)
   b) \(\frac{6x^2 - 13x + 6}{3x^2 - 2x} \div \frac{8x^3 - 27}{4x^3 + 6x^2 + 9x}\)

19. Perform the operations and simplify:
   a) \(\frac{4}{2x + 1} - \frac{x}{x + 2}\)
   b) \(\frac{2x}{x^2 + 1} - \frac{1}{x^3 + x}\)
   c) \(\frac{x + 1}{x - 1} - \left(\frac{x + 1}{x - 1}\right)^{-1}\)

20. Simplify each compound fraction. Find the domain of each fraction.
   a) \(\frac{x^2}{(x + 1)^2 + x}\)
   b) \(\frac{1 - 4}{x + 4} + \frac{6}{x^2}\)

21. Simplify completely:
   a) \(\frac{x^2 - y^2}{x - y}\)
   b) \(\frac{3x}{\sqrt{x^2 + 4} - \sqrt{x^2 + 4}}\)

22. Simplify the difference quotient:
   a) \(\frac{1}{x + h} - \frac{1}{x}\)
   b) \(\frac{\sqrt{x + h + 1} - \sqrt{x + 1}}{h}\) (rationalize the numerator)
23. Solve each equation:
   a) \( x - 3(2x + 3) = 9 - 5x \)   \( b) \left| 3 - 2x \right| + 4 = 2 \)

24. Solve each equation. Indicate all excluded values of the variable.
   a) \( \frac{3x}{8} - \frac{2x + 1}{4} = -1 \)   \( b) \frac{2}{x^2 - 6x + 8} - \frac{1}{x - 4} = \frac{2}{x - 2} \)  \( c) \frac{4y^3 + 12y^2}{y} = 0 \)

25. Solve the equation by completing the square, and check by solving using factoring: \( x^2 - 2x = 8 \)

26. Solve by completing the square: \( 9x^2 + 3 = 18x \)

27. Use the discriminant to determine the nature of the roots of each of the following quadratic equations:
   a) \( x(x - 2) = 2 \)   \( b) 2x^2 - 3x + 6 = 0 \)  \( c) 4x^2 + 9 = 12x \)

28. Solve each equation if possible using the quadratic formula:
   a) \( x(x-2) = 2 \)   \( b) 2x^2 - 3x + 6 = 0 \)  \( c) 4x^2 + 9 = 12x \)  \( d) \frac{3x}{x - 2} + \frac{1}{x} = 4 \)

29. Solve each equation and check your answers:
   a) \( \sqrt{2x + 5} + 3 = 0 \)   \( b) x = \sqrt{15 - 2x} \)  \( c) \sqrt{2x + 3} - \sqrt{x + 2} = 2 \)

30. Solve for \( x \) and check your solutions:
   a) \( (x + 6)^{3/2} = 8 \)   \( b) (3x - 2)^{2/3} = 9 \)

31. Solve the absolute value equations:
   a) \( |2x - 3| + 2 = 9 \)   \( b) |x^2 + 6x| = 3x + 18 \)  \( c) |2x + 3| = x \)

32. Solve the following equations by factoring or substitution:
   a) \( x^3 - x^2 + 4 = 4x \)   \( b) 4x^4 - 12x^2 + 9 = 0 \)
   \( c) (x^2 - 1)^2 + (x^2 - 1) - 12 = 0 \)  \( d) 6x^{-2} + x^{-1} = 2 \)

33. Find all excluded values of the variable and solve by factoring:
   \( 2(x + 1)^{-1/3}x^{4/3} - (x + 1)^{2/3}x^{-2/3} = 0 \)
   (Hint: write without negative exponents after factoring)
This review, produced by the CLAS Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:

You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/
1. Sketch the following subsets of the real numbers on a number line
   (a) $[-4, 12)$
   (b) $(-\infty, 0]$ 
   (c) $x - 3 \leq 5$
   (d) $x$ is no larger than 9.

2. Sketch the following subsets of the real numbers on a number line
   (a) $|x| < 2$
   (b) $|x| \geq 2$
   (c) $|x - 1| > 3$
   (d) The distance from $x$ to 1 is at least 2.

3. Simplify the radical expressions
   (a) $\frac{\sqrt{18z^6x^7}}{\sqrt{2z^3}}$
   (b) $\sqrt[5]{64x^{10} - 32x^5}$
   (c) $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{1^2 + 1^2}}{2}$

4. Write each function piecewise without absolute value bars.
   (a) $e(x) = |x|$
   (b) $f(x) = |2x + 1|$
   (c) $h(x) = \frac{1}{|x|}$

5. Expand each of the following expressions.
   (a) $(x + 1)^2$
   (b) $(x - 2)(x + 2)$
   (c) $(2x - 1)^3$
   (d) $(x + y + 1)(x + y - 1)$
6. Determine the requested coefficient in the expanded form of each expression below.

(a) Coefficient of $x^2$ in $(2x - 4)(1 - x)$
(b) Coefficient of $x$ in $(2x + 1)^2$
(c) Coefficient of $x^3$ in $(2x + 3)^3$
(d) Coefficient of $xy$ in $(x + y - 1)(2x - 3y + 2)$

7. Factor each of the following polynomials.

(a) $8y^2(x + 3) - 2(x + 3)$
(b) $x^4 - 4$
(c) $x^2 + x - 6$
(d) $3x^2 - 6x + 3$
(e) $2x^2 + 5x - 3$
(f) $x^3 - 27$
(g) $x^3 + 3x^2 - 6x - 18$

8. Simplify each expression, leaving positive exponents only.

(a) $3x^{-4/3} + 2x^{-1/3}$
(b) $-x^{-1}(1 + x^2)^{-2/3} - 2x^{-3}(1 + x^2)^{1/3}$
(c) $x^2(1 - 2x)^{-3/2} + (1 - 2x)^{-1/2}$
(d) $\frac{-2(x^2 - 3)^{-3}(2x)(x + 1)^3 - 3(x + 1)^2(x^2 - 3)^{-2}}{[(x + 1)^3]^2}$

9. Solve the following equations for the indicated variable.

(a) $y = \frac{x + 2}{x - 1}$ for $x$.
(b) $4x^2 - 1 = 7$ for $x$
(c) $\sqrt{3 - 2t} = t$ for $t$
(d) $|x - 2| = 1$ for $x$
(e) $x + \sqrt{31 - 9x} = 5$ for $x$
(f) $\frac{10x + 3}{5x + 6} = \frac{1}{2}$ for $x$
(g) $(x^2 - x - 22)^{3/2} = 27$ for $x$
1) Each set of numbers on the left can be described as being an example of a kind of number on the right. Match each set on the left to the kind of the number that best describes its members on the right.

<table>
<thead>
<tr>
<th>Set</th>
<th>Kind of Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>{\sqrt{2}, \pi, \sqrt{\pi}}</td>
<td>Real</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td>Whole</td>
</tr>
<tr>
<td>{77, -24, \sqrt{5}}</td>
<td>Natural</td>
</tr>
<tr>
<td>{-3, 4, 0}</td>
<td>Irrational</td>
</tr>
<tr>
<td>{0, 1, 2}</td>
<td>Integer</td>
</tr>
</tbody>
</table>

2) Write without absolute value bars and simplify:
   a) \(|\pi - 7| - |4 - \pi|\)  
   b) \(|10\pi - 32| / (5\pi - 16)\)

3) Find the products:
   a) \((2x + 3y - 1)(2x - 3y - 1)\)  
   b) \((2x + 3y)(2x - 3y)(4x^2 + 9y^2)\)

4) Factor completely. Leave the expression rationalized and with only positive exponents:
   a) \(7(3x^2 + 2)^2(1 - x^2)^2 + (3x^2 + 2)(1 - x^2)^3\)  
   b) \(-x^{-1}(1 + x^2)^{-2} - 2x^{-3}(1 + x^2)^{\frac{1}{2}}\)

5) Simplify the rational expressions.
   a) \(\frac{x^3 - 8}{x^2 - 4} + \frac{x^3 + 2x^2 + 4x}{x^2 + x - 2}\)  
   b) \(\frac{(x + 1)^2(2x) - x^2(2)(x + 1)}{(x + 1)^4}\)

6) Simplify:
   a) \(\frac{x^2y}{4z^{-1}}(8x^{-2}yz^3)\)  
   b) \(\frac{3\sqrt[3]{16x^2y^{-6}z^{1.0}}}{y^{-5}z^{-2}}\)  
   c) \(\sqrt[3]{64}\)  
   d) \(\sqrt[3]{320a^7bc^6}\)

7) Simplify:
   a) \(\frac{4 - 2\sqrt{3} + 3}{2 - \sqrt{3}}\)  
   b) \(\sqrt[3]{8} + \frac{1}{\sqrt{2}}\)

8) Solve and check the solutions:
    \(2\sqrt{x + 1} - \sqrt{2x + 3} = 1\)

9) Solve for \(y\) in ...
    \(\frac{3}{y - 2} + \frac{1}{y + 1} = \frac{1}{y^2 - y - 2}\)