A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 2, 1.
   1  ●  3  4  5  6  7  8  9  0
   ●  2  3  4  5  6  7  8  9  0

D. At the top right of your answer sheet, for “Test Form Code”, encode A.
   ●  B  C  D  E

E. 1) This test consists of 9 five-point multiple choice questions and two pages (both sides) of free reponse questions worth 40 points. So you can earn up to 85 points out of 80 on this exam.
   2) The time allowed is 90 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:
   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
   2) Bring your test, scratch paper, bubble sheet, and any tearoff sheets to your proctor to turn them in. Be prepared to show your UF ID card.
   3) Answers will be posted in Canvas after the test.
1. Minimize the function \( f(x, y) = x^2 + 16y^2 \) subject to the constraint \( xy = 3 \).

A. 24  B. 22  C. \( 25\sqrt{3} \)  D. 25  E. \( 22\sqrt{2} \)

2. The total weekly revenue (in dollars) of the Country Workshop realized in manufacturing and it’s rolltop desks is given by

\[
R(x, y) = -0.2x^2 - 0.25y^2 - 0.2xy + 196x + 175y
\]

where \( x \) denotes the number finished units and \( y \) denotes the number of unfinished units manufactured and sold each week. The total weekly cost attributable to the manufacture of these desks is given by

\[
C(x, y) = 70x + 60y + 4000
\]

dollars. Determine how many finished units and how many unfinished units the company should manufacture each week in order to maximize its profit. What is the maximum profit (\( P \)) realizable?

A. \((250, 130); \quad P = \$19,470\)
B. \((130, 250); \quad P = \$19,470\)
C. \((380, 0); \quad P = \$19,225\)
D. \((250, 130); \quad P = \$19,225\)
E. \((130, 250); \quad P = \$19,970\)

3. Let \( f(x, y) = x^2 \ln(y) \). Compute \( f_x(1, e) - f_y(2, 4) + f_{yy}(3, 1) \).

A. -8  B. 10  C. -6  D. 12  E. -4

4. Find the approximate change in \( f(x, y) = \frac{x - y}{x + y} \) when the point \((x, y)\) changes from \((-3, -2)\) to \((-3.02, -1.98)\).

A. 0.004  B. 0.008  C. 0.016  D. -0.8  E. 0.8
5. The production of a certain company is given by the function

\[ f(x, y) = 100x^{3/4}y^{1/4} \]

when \( x \) units of labor and \( y \) units of capital are utilized. Find the approximate percentage change in the production of the company if labor is increased by 4% and capital is increased by 8%.

A. 6%  
B. 7%  
C. 8%  
D. 4%  
E. 5%

6. Evaluate the double integral \( \iint_{\mathcal{R}} 6x^2y \, dA \) where \( \mathcal{R} \) is the rectangle defined by \(-1 \leq x \leq 2\) and \(0 \leq y \leq 1\).

A. \( \frac{7}{2} \)  
B. 6  
C. 9  
D. \( \frac{9}{2} \)  
E. 3

7. Suppose that \( f(x, y) \) is continuous and differentiable everywhere. Suppose that \( f_x(3, 2) = 0 \) and \( f_y(3, 2) = 0 \). Also, suppose that \( f_{xx}(3, 2) = 3 \), \( f_{yy}(3, 2) = 3 \) and \( f_{xy}(3, 2) = -3 \). Then what does the second derivative test say about at \( f(x, y) \) at (3, 2)?

A. Relative Maximum at (3, 2)  
B. Relative Minimum at (3, 2)  
C. Saddle Point at (3, 2)  
D. Second Derivative Test is inconclusive at (3, 2)  
E. No critical points to check

8. Which of the following statements is/are true:

I. If \( f_x(x, y) \) is defined at (1, 1), then \( f_y(x, y) \) must also be defined at (1, 1).

II. If \( f(x, y) \) is continuous at (1, 1) then \( f_x(1, 1) \) must exist.

III. Let \( f(x, y) = x^2 \), then \( f(x, y) \) has an infinite number of critical points.

A. I only  
B. II only  
C. I and III  
D. I, II, and III  
E. III only

9. Use the region \( R \) with indicated boundaries to evaluate \( \iint_{R} 5x^2y \, dA \) where \( R \) is bounded by \( y = x \) and \( y = x^2 \).

A. \( \frac{2}{7} \)  
B. \(-\frac{1}{7}\)  
C. \( \frac{1}{7} \)  
D. \( \frac{3}{7} \)  
E. \(-\frac{2}{7}\)
Reverse the order of the integration \( \int_{0}^{1} \int_{\sqrt{y} / 2}^{1} e^{-x^2} \, dx \, dy \). Evaluate the integral with the order reversed and find the \textbf{exact} answer. Do not attempt to evaluate the integral in the original form. Be sure to sketch the region you are integrating over.
2. Sketch the level curves for \( f(x, y) = e^{y - \sqrt{x}} \) with \( z = 1, e, e^2 \). Be sure to label each of the level curves.

3. Sketch the domain of \( f(x, y) = \sqrt{x^2 - y} \). Sketch the domain, shade in the region and label the axes.
4. Classify all extrema for \( f(x, y) = 4y^3 + x^2 - 12y^2 - 36y + 2 \).

Fill in the blanks (if none, write “none”):

(a) Relative maximum with \( x = \) _______ and \( y = \) _______

(b) Relative maximum value \( z = \) _______

(c) Relative minimum with \( x = \) _______ and \( y = \) _______

(d) Relative minimum value \( z = \) _______

(e) Saddle point \( x = \) _______ and \( y = \) _______
5. (a) Find the best fit curve of the form \( y = ax^2 + 1 \) that passes through the points \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\). Find a formula for \( a \).

(b) (Bonus) Prove that this value of \( a \) minimizes the distance between the curve \( y = ax^2 + 1 \) and the data points. (Hint: consider the second derivative.)

(c) Use the formula you constructed from part (a) to find the value of the best fit curve \( y = ax^2 + 1 \) that passes through \((1, 3), (2, 4), (3, 2)\).