Disclaimer: This review is by no means complete. Please review the textbook chapter 8.

1. Find the domain of each function:
   (a) \( f(x, y) = \ln(x + 3y) \)
   (b) \( g(x, y) = \frac{xy}{x^2 - y^2} \)
   (c) \( h(x, y) = \sqrt{16 - x^2 - y^2} \)

2. Sketch the level curves of the functions corresponding to each \( z \) values.
   (a) \( f(x, y) = e^x - y \) with \( z = -2, -1, 0, 1, 2 \).
   (b) \( g(x, y) = xy \) with \( z = -2, -1, 1, 2 \).
   (c) \( h(x, y) = \ln(x - y) \) with \( z = -2, -1, 0, 1, 2 \)

3. Let \( f(x, y) = x^2 - xy + 5y^2 \). Compute
   \[ \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}. \]
   What is the geometric interpretation of these limits?

4. Let \( f(x, y) = x^3 \ln(y) + 4y^2 e^x \). Find indicated function or value.
   - \( f_y(x, y) \)
   - \( f_x(x, y) \)
   - \( f_{yy}(x, y) \)
   - \( f_{yx}(x, y) \)
   - \( f_{xx}(x, y) \)
   - \( f_{xy}(x, y) \)
   - \( f_{yx}(-1, 1) \)
   - \( f_{yy}(-1, 1) \)

5. Find the critical points of the function then use the second derivative test to classify the nature of each point. Determine the relative extrema of the function.
   (a) \( f(x, y) = x^3 + y^2 - 6xy. \)
   (b) \( g(x, y) = e^{x^2+y^2} \)
   (c) \( h(x, y) = xy + \ln x + 2y^2 \)

6. Explain why \( f(x, y) = x^2 \) has an infinite number of local extrema.

7. Show that \( f(x, y) = \sqrt{x^2 + y^2} \) has one critical point \( P \) and that \( f \) is nondifferentiable at \( P \). Does \( f \) take on a minimum, maximum or saddle point at \( P \)?
8. A firm produces two types of earphones per year: \( x \) thousand of type A and \( y \) thousand of type B. If the revenue and cost equations for the year (in millions of dollars)

\[
R(x, y) = 2x + 3y \\
C(x, y) = x^2 - 2xy + 2y^2 + 6x - 9y + 5
\]

determine how many of each type of earphone should be produced per year to maximize profit. What is the maximum profit?

9. Consider the data set \( D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\} \).

(a) Derive the formula for \( a \) for the least squares curve \( y = ax - 1 \) that best fits the data set \( D \).

(b) Derive the formula for \( b \) for the least squares curve \( y = 2x + b \) that best fits the data set \( D \).

(c) Use the formulas in the previous parts on the data set \( \{(1, 3), (2, 1), (3, 2), (4, 0)\} \).

10. Use the method of Lagrange multipliers to minimize or maximize the function with the constraint. Explain how you know it’s a max or a min.

(a) \( f(x, y) = 3x^2 + 5y^2 \) subject to \( 2x + 3y = 6 \).

(b) \( f(x, y) = xy \) subject to \( 3x + y = 720 \).

(c) \( f(x, y) = 2x^2 + y^2 + 2 \) subject to \( x^2 + 4y^2 = 4 \).

(d) \( f(x, y) = e^{3x-5y} \) subject to \( x^2 + y^2 = 1 \).

11. Let \( f(x, y) = xe^{xy} \) and suppose \((x, y)\) changes from \((1, 0)\) to \((0.9, 0.01)\). Compute \( dz \) and \( \Delta z \). Compare the values of \( \Delta z \) and \( dz \). How close is the approximation.

12. Find the total differential of the function \( f(x, y) = (x^2 + y^4)^{3/2} \) at the given point \((3, 2)\).

13. The price-earnings ratio (PE ratio) of a stock is given by

\[
R(x, y) = \frac{x}{y}
\]

where \( x \) denotes the price per share of the stock and \( y \) denotes the earnings per share. Estimate the change in the PE ratio \( R \) of a stock if its price increases from $62/share to $65/share while its earnings decrease from $3/share to $2.60/share. (Round your answer to two decimal places.)

Answer: 3.76.

14. Evaluate the double integral \( \int \int_{R} f(x, y) \, dA \) where \( f(x, y) = y + 2x \) and \( R \) is the rectangle defined by \( 1 \leq x \leq 2 \) and \( 0 \leq y \leq 1 \).
15. Evaluate the double integral $\int\int_{R} 2xy \, dA$ and $R$ is the region bounded by the graphs of $y = -x$ and $y = x^2$, $x \geq 0$ and $x = 1$. 

16. Find the volume of the solid bounded above by the surface $z = f(x, y) = 2x + y$ and below by the plane region $R$ where $R$ is the triangle bounded by $y = 2x$, $y = 0$, and $x = 3$. Answer: 54 cubic inches.

17. Reverse the order of integration for each integral. Evaluate the integral with the ordered reverse. Do not attempt to evaluate the integral in the original form.

(a) $\int_{0}^{2} \int_{x^2}^{4} \frac{4x}{1 + y^2} \, dy \, dx$

(b) $\int_{0}^{1} \int_{y}^{1} \sqrt{1 - x^2} \, dx \, dy$

18. The production of a certain company is given by the function

$$f(x, y) = 50x^{1/3}y^{2/3}$$

when $x$ units of labor and $y$ units of capital are utilized. Find the approximate percentage change in the production of the company if labor is increased by 6% and capital is increased by 5%. (Round your answer to two decimal places.) (Ans: 5.33%)