A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number (The section number for this class is 1936)

C. Under “special codes”, code in the test ID number 4, 1.

   1 2 3 ● 5 6 7 8 9 0
   ● 2 3 4 5 6 7 8 9 0

D. At the top right of your bubble sheet, for “Test Form Code”, encode A.

   ● B C D E

E. 1) This test consists of 22 five-point multiple choice questions worth 110 points
    (100 points plus 10 bonus points).
   2) The time allowed is 120 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper or if you have a problem with your
      test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE
      TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in
      are there to stay.
   2) You must turn in your scantron to the proctor of the exam. You may keep your
      exam and scratch paper to check your answers. Be prepared to show your UF ID
      card.
   3) Answers will be posted in Canvas after the test. Your score will be posted in the
      Canvas gradebook within one week of the exam.
1. Let $Z$ be the standard normal random variable. Calculate $P(-1.5 < Z < 2)$. Use the standard normal distribution table on the last page of the exam.

A. 0.9224  B. 0.9045  C. 0.8964  D. 0.9153  E. 0.9104

2. Consider $A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Determine which of the following statements are true, if any.

I. $A$ is a matrix of size $3 \times 4$.

II. $A$ is in row-reduced form.

III. If $B$ is a $3 \times 3$ matrix, then $AB$ will be a $3 \times 4$ matrix.

A. None  B. I only  C. III only  D. I and II only  E. I, II, and III

3. Evaluate $\int_0^1 \int_{\sqrt{\pi}}^1 3e^{y^3} \, dy \, dx$ by reversing the order of integration. Do not attempt to evaluate the double integral in its current form.

A. $1 + e$  B. $1 + e^3$  C. $e - 1$  D. $e^3 - 1$  E. $1 - e$
4. Given \( a_n = \frac{1}{4^n} \), determine the convergence or divergence of both \( \{a_n\} \) and \( \sum_{n=0}^{\infty} a_n \).

A. \( \{a_n\} \) diverges and \( \sum_{n=0}^{\infty} a_n \) converges.

B. \( \{a_n\} \) converges and \( \sum_{n=0}^{\infty} a_n \) diverges.

C. \( \{a_n\} \) and \( \sum_{n=0}^{\infty} a_n \) both diverge.

D. \( \{a_n\} \) and \( \sum_{n=0}^{\infty} a_n \) both converge.

E. The convergence or divergence of \( \{a_n\} \) cannot be determined.

5. A clothes shop determines the quantity of sweaters \( x \) demanded per week is related to the unit price \( p \) (in dollars) by \( p = 100 - 2x \). Determine the consumers’ surplus if the selling price is set to $60.

A. $3600  
B. $1200  
C. $2400  
D. $1600  
E. $400

6. Approximate \( \int_0^3 \frac{20}{x^2 + 1} \, dx \) using the Trapezoidal Rule with \( n = 3 \).

A. 25  
B. 30  
C. 24  
D. 15  
E. 20
7. Determine the second degree Taylor polynomial of \( f(x) = \frac{4}{x} \) centered at \( x = 1 \).

A. \( \frac{4}{x} + 4 \left( \frac{x-1}{x} \right) - 8 \left( \frac{x-1}{x} \right)^2 \)
B. \( 4 - 4(x-1) + 8(x-1)^2 \)
C. \( 4 - 4(x-1) + 4(x-1)^2 \)
D. \( 4 + 4(x-1) - 4(x-1)^2 \)
E. \( 4 + 4(x-1) - 8(x-1)^2 \)

8. Determine the area of the region bounded by the curves \( y = 3xe^{-3x} \), \( y = 0 \), \( x = 0 \), and \( x = 1 \).

A. \( \frac{1 - 4e^{-3}}{3} \)
B. \( \frac{1 + 4e^{-3}}{3} \)
C. \( \frac{4e^{-3}}{3} \)
D. \( 1 - 4e^{-3} \)
E. \( 1 + 4e^{-3} \)

9. Consider the function \( y = 2t^3 \). Determine which of the following differential equations have \( y \) as a solution, if any.

I. \( y' + y = 2t^3 \)
II. \( ty' = 3y \)
III. \( y'' - y = 0 \)

A. None
B. I only
C. II only
D. II and III only
E. I, II, and III
10. Given the augmented matrix below, determine the solution to the corresponding system of equations.

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 2 & 3
\end{bmatrix}
\]

A. Unique solution; (4, 3)

B. Infinitely many solutions; (4, 1 + 2t + 3, t)

C. Infinitely many solutions; (s, t, 4t + 3s)

D. Infinitely many solutions; (4, 3 − 2t, t)

E. No solution

11. The function \( f(x, y) \) has critical points at (1, 2) and (3, 1). The following information is known about \( f(x, y) \).

\[
\begin{align*}
f_{xx}(1, 2) &= 2 & f_{yy}(1, 2) &= 3 & f_{xy}(1, 2) &= 1 \\
f_{xx}(3, 1) &= -1 & f_{yy}(3, 1) &= 2 & f_{xy}(3, 1) &= -2
\end{align*}
\]

Determine which one of the following statements is true.

A. \( f \) has a relative minimum at (1, 2) and a saddle point at (3, 1).

B. \( f \) has a relative maximum at (1, 2) and a saddle point at (3, 1).

C. \( f \) has a saddle point at (1, 2) and a relative minimum at (3, 1).

D. \( f \) has a relative minimum at (1, 2) and a relative maximum at (3, 1).

E. \( f \) has a relative maximum at (1, 2) and a relative maximum at (3, 1).
12. Determine which one of the following statements is true.

A. \(\int_{1}^{\infty} \frac{2}{x^2} \, dx\) converges since \(\lim_{b \to \infty} \frac{2}{x^2} \bigg|_{1}^{b} = \lim_{b \to \infty} \left( \frac{2}{b^2} - 2 \right) = -2\) converges.

B. \(\int_{1}^{\infty} \frac{2}{x^2} \, dx\) diverges since \(\lim_{b \to \infty} \frac{2}{x^2} \bigg|_{1}^{b} = \lim_{b \to \infty} \left( \frac{2}{b^2} - 2 \right) = \infty\) diverges.

C. \(\int_{1}^{\infty} \frac{2}{x^2} \, dx\) converges since \(\lim_{b \to \infty} \frac{-2}{x} \bigg|_{1}^{b} = \lim_{b \to \infty} \left( 2 - \frac{2}{b} \right) = \infty\) converges.

D. \(\int_{1}^{\infty} \frac{2}{x^2} \, dx\) converges since \(\lim_{b \to \infty} \frac{-2}{x} \bigg|_{1}^{b} = \lim_{b \to \infty} \left( 2 - \frac{2}{b} \right) = 2\) converges.

E. \(\int_{1}^{\infty} \frac{2}{x^2} \, dx\) diverges since \(\lim_{b \to \infty} \frac{-2}{x} \bigg|_{1}^{b} = \lim_{b \to \infty} \left( 2 - \frac{2}{b} \right) = \infty\) diverges.

13. Given \(A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\) and \(B = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}\), calculate \(A^{-1}B\).

A. \(\begin{bmatrix} 3 & 13 \\ 1 & 3 \end{bmatrix}\)  
B. \(\begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}\)  
C. \(\begin{bmatrix} 7 & -3 \\ 1 & -1 \end{bmatrix}\)

D. \(\begin{bmatrix} 0.25 & 1.25 \\ 0.25 & 0.25 \end{bmatrix}\)  
E. \(\begin{bmatrix} 3 & -11 \\ 1 & -5 \end{bmatrix}\)

14. Let \(f(x, y) = \frac{5x}{y^2} + \frac{1}{y}\) and suppose \((x, y)\) changes from \((2, 1)\) to \((2.02, 1.01)\).

Approximate the change in \(f\) using differentials.

A. 0.31  
B. -0.01  
C. -0.11  
D. 0.01  
E. 0.11
15. The method of Lagrange multipliers is used to determine the relative extrema of 
\[ f(x, y) = x^2 + 2y \] subject to the constraint \( x^2 + y^2 = 9 \). The candidates for the relative 
extrema are found to be
\[(0, -3), (0, 3), (-\sqrt{8}, 1), \text{ and } (\sqrt{8}, 1).\]

Determine which one of the following statements is true.

A. \( f \) has a relative maximum at \((\sqrt{8}, 1)\) and a relative minimum at \((0, 3)\).
B. \( f \) has a relative minimum at \((-\sqrt{8}, 1)\) and a relative maximum at \((\sqrt{8}, 1)\).
C. \( f \) has a relative maximum at \((0, 3)\) and a relative minimum at \((0, -3)\).
D. \( f \) has a relative maximum at \((-\sqrt{8}, 1)\) and a relative minimum at \((0, -3)\).
E. \( f \) has a relative maximum at \((0, 3)\) and a relative minimum at \((-\sqrt{8}, 1)\).

16. The random variable \( X \) is associated with the probability density function \( f(x) = 7x^6 \) 
where \( 0 \leq x \leq 1 \). Determine the mean of \( X \).

A. \( \frac{7}{6} \)  B. \( \frac{7}{9} \)  C. \( \frac{7}{576} \)  D. 1  E. \( \frac{7}{8} \)

17. Determine the average value of \( f(x) = \frac{2x}{\sqrt{x^2 + 9}} \) on \([0, 4]\).

A. 16  B. \( \frac{1}{4} \)  C. \( \frac{\ln(4)}{4} \)  D. 1  E. 4

18. Let \( f(x, y) = \frac{y^4}{4} - \frac{y^2}{2} + 4x^2 \). Determine which of the following are critical points of \( f \).

I. \((0, 0)\)  II. \((0, 1)\)  III. \((0, -1)\)

A. I only  B. II only  C. I and II only
D. II and III only  E. I, II, and III
19. The simplex method is used to solve a standard maximization problem. The following matrix results at the end of the process. Determine the optimal solution.

\[
\begin{bmatrix}
3 & 0 & 1 & 2 & 0 & 2 \\
1 & 1 & 0 & 3 & 0 & 4 \\
2 & 0 & 0 & 1 & 1 & 3 \\
\end{bmatrix}
\]

A. \(x = 2, \; y = 4, \; P = 7\)       B. \(x = 0, \; y = 4, \; P = 3\)       C. \(x = 2, \; y = 4, \; P = 3\)

D. \(x = 2, \; y = 3, \; P = 7\)       E. \(x = 0, \; y = 4, \; P = 7\)

20. Solve \(\frac{dy}{dx} = \frac{4\sqrt{x}}{y^2}\) where \(y(0) = 1\).

A. \(y^3 = 8x^{3/2} + 1\)       B. \(y^3 = 8x^{3/2} - 8\)       C. \(y^3 = 16x^{3/2} + 1\)

D. \(y^3 = 16x^{3/2} - 8\)       E. \(y^2 = 16x^{3/2} + 1\)

21. A simplified economy of two industries has Leontief input-output matrix \(A\) and total output matrix \(X\), given below. Determine the consumer demand \(D\).

\[
A = \begin{bmatrix}
0.2 & 0.1 \\
0.3 & 0.4 \\
\end{bmatrix} \quad X = \begin{bmatrix}
20 \\
30 \\
\end{bmatrix}
\]

A. \(\begin{bmatrix} 2 \\ 23 \end{bmatrix}\)       B. \(\begin{bmatrix} 27 \\ 48 \end{bmatrix}\)       C. \(\begin{bmatrix} 13 \\ 12 \end{bmatrix}\)

D. \(\begin{bmatrix} 20 \\ 30 \end{bmatrix}\)       E. \(\begin{bmatrix} 7 \\ 18 \end{bmatrix}\)

22. Maximize \(P = 10x + 5y\) subject to the following constraints.

\[
\begin{align*}
10y + 8x & \leq 80 \\
x & \leq 5 \\
x & \geq 0, \; y \geq 0
\end{align*}
\]

A. \(P = 40\)       B. \(P = 100\)       C. \(P = 70\)       D. \(P = 50\)

E. \(P = 0\)

7A
The Standard Normal Distribution

\[
\begin{array}{cccccccccccc}
   z & 0.00 & 0.01 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 \\
0.0 & 0.5000 & 0.5040 & 0.5080 & 0.5120 & 0.5160 & 0.5199 & 0.5239 & 0.5279 & 0.5319 & 0.5359 \\
0.1 & 0.5398 & 0.5438 & 0.5478 & 0.5517 & 0.5557 & 0.5596 & 0.5636 & 0.5675 & 0.5714 & 0.5753 \\
0.2 & 0.5793 & 0.5832 & 0.5871 & 0.5910 & 0.5948 & 0.5987 & 0.6026 & 0.6064 & 0.6103 & 0.6141 \\
0.3 & 0.6179 & 0.6217 & 0.6255 & 0.6293 & 0.6331 & 0.6368 & 0.6406 & 0.6443 & 0.6480 & 0.6517 \\
0.4 & 0.6554 & 0.6591 & 0.6628 & 0.6664 & 0.6700 & 0.6736 & 0.6772 & 0.6808 & 0.6844 & 0.6879 \\
0.5 & 0.6915 & 0.6950 & 0.6985 & 0.7019 & 0.7054 & 0.7088 & 0.7123 & 0.7157 & 0.7190 & 0.7224 \\
0.6 & 0.7257 & 0.7291 & 0.7324 & 0.7357 & 0.7389 & 0.7422 & 0.7454 & 0.7486 & 0.7517 & 0.7549 \\
0.7 & 0.7580 & 0.7611 & 0.7642 & 0.7673 & 0.7703 & 0.7734 & 0.7764 & 0.7794 & 0.7823 & 0.7852 \\
0.8 & 0.7881 & 0.7910 & 0.7939 & 0.7967 & 0.7995 & 0.8023 & 0.8051 & 0.8078 & 0.8106 & 0.8133 \\
0.9 & 0.8159 & 0.8186 & 0.8212 & 0.8238 & 0.8264 & 0.8289 & 0.8315 & 0.8340 & 0.8365 & 0.8389 \\
1.0 & 0.8413 & 0.8438 & 0.8461 & 0.8485 & 0.8508 & 0.8531 & 0.8554 & 0.8577 & 0.8599 & 0.8621 \\
1.1 & 0.8643 & 0.8665 & 0.8686 & 0.8708 & 0.8729 & 0.8749 & 0.8770 & 0.8790 & 0.8810 & 0.8830 \\
1.2 & 0.8849 & 0.8869 & 0.8888 & 0.8907 & 0.8925 & 0.8944 & 0.8962 & 0.8980 & 0.8997 & 0.9015 \\
1.3 & 0.9032 & 0.9049 & 0.9066 & 0.9082 & 0.9099 & 0.9115 & 0.9131 & 0.9147 & 0.9162 & 0.9177 \\
1.4 & 0.9192 & 0.9207 & 0.9222 & 0.9236 & 0.9251 & 0.9265 & 0.9279 & 0.9292 & 0.9306 & 0.9319 \\
1.5 & 0.9322 & 0.9345 & 0.9357 & 0.9370 & 0.9382 & 0.9394 & 0.9406 & 0.9418 & 0.9429 & 0.9441 \\
1.6 & 0.9452 & 0.9463 & 0.9474 & 0.9484 & 0.9495 & 0.9505 & 0.9515 & 0.9525 & 0.9535 & 0.9545 \\
1.7 & 0.9554 & 0.9564 & 0.9573 & 0.9582 & 0.9591 & 0.9599 & 0.9608 & 0.9616 & 0.9625 & 0.9633 \\
1.8 & 0.9641 & 0.9649 & 0.9656 & 0.9664 & 0.9671 & 0.9678 & 0.9686 & 0.9693 & 0.9699 & 0.9706 \\
1.9 & 0.9713 & 0.9719 & 0.9726 & 0.9732 & 0.9738 & 0.9744 & 0.9750 & 0.9756 & 0.9761 & 0.9767 \\
2.0 & 0.9772 & 0.9778 & 0.9783 & 0.9788 & 0.9793 & 0.9798 & 0.9803 & 0.9808 & 0.9812 & 0.9817 \\
2.1 & 0.9821 & 0.9826 & 0.9830 & 0.9834 & 0.9838 & 0.9842 & 0.9846 & 0.9850 & 0.9854 & 0.9857 \\
2.2 & 0.9861 & 0.9864 & 0.9868 & 0.9871 & 0.9875 & 0.9878 & 0.9881 & 0.9884 & 0.9887 & 0.9890 \\
2.3 & 0.9893 & 0.9896 & 0.9898 & 0.9901 & 0.9904 & 0.9906 & 0.9909 & 0.9911 & 0.9913 & 0.9916 \\
2.4 & 0.9918 & 0.9920 & 0.9922 & 0.9925 & 0.9927 & 0.9929 & 0.9931 & 0.9932 & 0.9934 & 0.9936 \\
2.5 & 0.9938 & 0.9940 & 0.9941 & 0.9943 & 0.9945 & 0.9946 & 0.9948 & 0.9949 & 0.9951 & 0.9952 \\
2.6 & 0.9953 & 0.9955 & 0.9956 & 0.9957 & 0.9959 & 0.9960 & 0.9961 & 0.9962 & 0.9963 & 0.9964 \\
2.7 & 0.9965 & 0.9966 & 0.9967 & 0.9968 & 0.9969 & 0.9970 & 0.9971 & 0.9972 & 0.9973 & 0.9974 \\
2.8 & 0.9974 & 0.9975 & 0.9976 & 0.9977 & 0.9977 & 0.9978 & 0.9979 & 0.9979 & 0.9980 & 0.9981 \\
2.9 & 0.9981 & 0.9982 & 0.9982 & 0.9983 & 0.9984 & 0.9984 & 0.9985 & 0.9985 & 0.9986 & 0.9986 \\
3.0 & 0.9987 & 0.9987 & 0.9987 & 0.9988 & 0.9988 & 0.9989 & 0.9989 & 0.9989 & 0.9990 & 0.9990 \\
\end{array}
\]