It is your responsibility to be sure that your test has **17 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

**Part I Instructions**: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find a $c$ value that satisfies the Mean Value Theorem for the function $f(x) = x^2 + 4$ on the interval $[0, 4]$.

   (A) 0  (B) 8  (C) 2  (D) 4  (E) Does not satisfy hypotheses of MVT

2. If $f(x)$ is continuous and differentiable, $f(-5) = 2$, and $f'(x) \geq -2$, what is the least possible value of $f(5)$?

   (A) -18  (B) -8  (C) 2  (D) 22  (E) No limit to how small $f(5)$ can be
3. To which of the following function/interval pairs can Rolle’s Theorem be applied?

   (i) \( f(x) = x^2 - 2x + 3, \ [0, 2] \)
   (ii) \( g(x) = \frac{1}{x^2}, \ [-1, 1] \)
   (iii) \( h(x) = \sin(x) - 1, \ [\pi/6, 11\pi/6] \)

   (A) (i)  (B) (i) and (ii)  (C) (i) and (iii)  (D) (ii) and (iii)  (E) (i), (ii), and (iii)

4. Suppose \( f''(x) \) is continuous on \((-\infty, \infty)\). If \( f'(7) = 0 \) and \( f''(7) = -2 \), what can you say about \( f(x) \)?

   (A) \( f(x) \) has a local minimum at \( x = 7 \)
   (B) \( f(x) \) has a local maximum at \( x = 7 \)
   (C) \( f(x) \) has neither a maximum nor minimum at \( x = 7 \)
   (D) More information is needed to determine if \( f(x) \) has a max/min at \( x = 7 \)

5. Consider the function \( f(x) = \ln(x^2 - 25) \). At \( x = 0 \), \( f(x) \) has

   (A) a local maximum  (B) a local minimum  (C) an inflection point  (D) \( x = 0 \) is not in the domain
6. The graph of a function \( f(x) \) is given above. Which statement out of the following is true?

(A) \( f'(x) \) is positive on \((0, \infty)\)
(B) \( f(x) \) has exactly three critical values
(C) \( f''(x) \geq 0 \) for all values of \( x \) where \( f''(x) \) exists
(D) \( f'(0.5) \) and \( f''(0.5) \) have the same sign

7. Let \( g(x) \) be a function such that \( g'(x) \) is positive on \((0,1)\), \( g''(x) \) is negative on \((-1,1)\), and \( \lim_{x \to \infty} g(x) = 2 \). Which of the following could be the graph of \( g(x) \)?

(A) 
(B) 
(C) 
(D)
8. Evaluate \( \lim_{x \to 0} \cot(2x) \sin(6x) \).

\[ (A) -\infty \] \hspace{1cm} \[ (B) 0 \] \hspace{1cm} \[ (C) 3 \] \hspace{1cm} \[ (D) \infty \] \hspace{1cm} \[ (E) \text{None of the above} \]

9. Evaluate \( \lim_{x \to 1} \frac{\sin(\pi x)}{\ln(x)} \).

\[ (A) 0 \] \hspace{1cm} \[ (B) 1 \] \hspace{1cm} \[ (C) \pi \] \hspace{1cm} \[ (D) \infty \] \hspace{1cm} \[ (E) \text{None of the above} \]

10. The volume of a cube with side lengths \( x \) is \( V = x^3 \). If the cube expands as time passes, which of the following gives \( \frac{dV}{dt} \) in terms of \( \frac{dx}{dt} \)?

\[ (A) \frac{dV}{dt} = x^3 \frac{dx}{dt} \] \hspace{1cm} \[ (B) \frac{dV}{dt} = 3 \left( \frac{dx}{dt} \right)^2 \] \hspace{1cm} \[ (C) \frac{dV}{dt} = x^2 \frac{dx}{dt} \] \hspace{1cm} \[ (D) \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \]
11. If $e^x y = \cos(x)$ where both $x$ and $y$ are functions of $t$, what is the value of $\frac{dy}{dt}$ when $\frac{dx}{dt} = 2$ and $x = 0$?

   (A) 4  (B) 0  (C) 2  (D) -2

12. Evaluate $\lim_{x \to 1} \frac{x^2 + 4}{3x + 2}$

   (A) 1  (B) $\frac{2}{3}$  (C) $\infty$  (D) $\frac{1}{3}$  (E) $-\infty$
13. If \( f(x) = x \ln(x) \) and \( x \) goes from 1 to 1.3, then how many of the following are true?

(i) \( \Delta y = 1.3 \ln(1.3) \)

(ii) \( dy = 0.3 \)

(iii) \( dx = 0.3 \)

(iv) \( \Delta x = 0.3 \)

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

14. If \( f(x) = 2x^3 - 15x^2 + 24x + 7 \), then the absolute minimum of \( f(x) \) on \([0, 2]\) is equal to:

(A) 18  (B) −27  (C) −9  (D) 7  (E) −4
Part II Instructions: 3 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. A total of 35 points is possible on Part II. **No credit will given without proper work.** If we cannot read it and follow it, you will receive no credit for the problem.

For Instructor Use Only:

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1. Find the linearization of the function \( f(x) = (4x + 2)e^x \) at \( a = 1 \) and use it to approximate the value of \( f(1.1) \) (leave your answers in terms of the number \( e \)).

2. Two people start moving from the same point. Person \( A \) travels south at 2.5 m/s and Person \( B \) travels west at 6 m/s. At what rate is the distance between the two people increasing two seconds later?
3. (21pts) Consider the function $y = f(x)$ where

$$
\begin{align*}
  f(x) &= \frac{(x - 1)^3}{3x^2} \\
  f'(x) &= \frac{(x - 1)^2(x + 2)}{3x^3} \\
  f''(x) &= \frac{2(x - 1)}{x^4}
\end{align*}
$$

(a) What is the domain of $f(x)$? What are the vertical and horizontal asymptotes of $f(x)$?

(b) List the critical point(s) of $f(x)$ (write them as coordinate pairs $(x, y)$). On what interval(s) is $f(x)$ increasing? decreasing?

(c) List the point(s) of inflection (write them as coordinate pairs $(x, y)$). On what interval(s) if $f(x)$ concave up? concave down?
(d) At what point(s) does \( f(x) \) have a local maximum? local minimum? (Write them as coordinate pairs \((x, y)\))

(e) Sketch the graph of \( f(x) \). Label all asymptotes, local maximum, local minimum, and inflection points.