It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

**Part I Instructions**: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Which of the following is the equation for the tangent line to \( f(x) = x^2 + x \) at the point \((1,2)\)? Note that \( f'(x) = 2x + 1 \).

   \[
   (A) \ y = 3x - 1 \quad (B) \ y = 3x + 1 \quad (C) \ y = 3x - 5 \quad (D) \ y = 2x - 3 \quad (E) \ y = 2x - 1.
   \]

2. Consider the function \( f(t) = 3t^2 + 1 \). Then \( f'(t) = 6t \). What is the second derivative of \( f(t) \), i.e. find \( f''(t) \).

   \[
   (A) \ 3 \quad (B) \ 6 \quad (C) \ -6 \quad (D) \ -3 \quad (E) \ \text{None of the above}
   \]
3. What type of discontinuity does the function \( f(x) = \frac{x^2 - 3x - 10}{x+2} \) have at \( x = -2 \)?

(A) Infinite   (B) Jump   (C) Removable   (D) \( f(x) \) is continuous at \( x = -2 \)   (E) None of the above

4. Let \( f(x) = \frac{\ln(x)}{x^2} \). Using the definition of the derivative at a point, which of the following is equal to \( f'(1) \)?

(A) \( \lim_{h \to 0} \frac{\ln(h)}{h^3} \)   (B) \( \lim_{h \to 0} \left( \frac{\ln(h)}{h^2} - \frac{\ln(1)}{h} \right) \)

(C) \( \lim_{h \to 0} \frac{\ln(1 + h)}{h(1 + h)^2} \)   (D) \( \lim_{h \to 0} \left( \frac{\ln(1 + h)}{h(1 + h)^2} - \frac{\ln(h)}{h^3} \right) \)

(E) None of the above

5. Find \( \lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 3x + 2} \)

(A) \( \infty \)   (B) 0   (C) \( \frac{5}{2} \)   (D) 5   (E) None of the above
6. Which of the following limits gives the derivative, \( f'(x) \), of the function \( f(x) = \sqrt{x - 2} \)?

\[
(A) \lim_{h \to 0} \frac{\sqrt{x + h - 2} - \sqrt{x - 2}}{h} \\
(B) \lim_{h \to 0} \frac{\sqrt{x - 2} - \sqrt{x + h - 2}}{h} \\
(C) \lim_{h \to 0} \frac{h}{\sqrt{x - 2} - \sqrt{x + h - 2}} \\
(D) \lim_{h \to 0} \frac{h}{\sqrt{x + h - 2} - \sqrt{x - 2}}
\]

7. How many of the following functions must have a root between \(-1\) and 2?

(i) \( f(x) = x^2 + 1 \)
(ii) \( g(x) = e^x - 1 \)
(iii) \( h(x) = \cos(\pi x) \)
(iv) \( k(x) = x^2 + x + 2 \)

(A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) 2 \hspace{1cm} (D) 3 \hspace{1cm} (E) 4

8. Which of the following is(are) the horizontal asymptotes for the function \( f(x) = \frac{8x + 2}{\sqrt{4x^2 + x}} \)?

(A) \( y = -4 \) only \hspace{1cm} (B) \( y = 4 \) only \hspace{1cm} (C) \( y = 0 \) and \( y = 4 \) \hspace{1cm} (D) \( y = 0 \) only \hspace{1cm} (E) \( y = 4 \) and \( y = -4 \)
9. Find the interval(s) on which the function \( f(x) = \ln(\sqrt{x} - 2) \) is continuous.

(A) \((-∞, 2) \cup (2, ∞)\)  (B) \(2, ∞\)  (E) \((-∞, 0) \cup (0, 2)\)  (C) \((-∞, ∞)\)  (D) \([2, ∞)\)

10. Let \( f(x) = \begin{cases} 
1/(x - 1), & x < 0 \\
x^2 - 1, & 0 \leq x < 1 \\
x, & 1 < x
\end{cases} \). How many of the following are true?

(i) The function has an infinite discontinuity.

(ii) The function has a nonremovable discontinuity.

(iii) The function has a jump discontinuity.

(iv) The function has a removable discontinuity.

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

11. Let \( f(x) \) and \( g(x) \) be function such that \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) exists and are real numbers, and let \( n \) be a positive integer. How many of the following are always true?

(i) \( \lim_{x \to a} \frac{1}{f(x)g(x)} = \left[ \frac{1}{\lim_{x \to a} f(x)} \right] \left[ \frac{1}{\lim_{x \to a} g(x)} \right] \)

(ii) \( \lim_{x \to a} (f(x) - g(x)) = (\lim_{x \to a} f(x)) - (\lim_{x \to a} g(x)) \)

(iii) \( \lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)} \)

(iv) \( \lim_{x \to a} [2 + (3g(x))^2] = 2 + 3(\lim_{x \to a} g(x))^2 \)

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4
12. A particle which moves along a straight line has position function \( s(t) = t^2 - 2t \) for \( t \geq 0 \), where \( s(t) \) is given in meters and \( t \) is given in seconds. What is the average velocity in meters per second of the particle between times \( t = 1 \) and \( t = 2 \)?

\((A) -1 \quad (B) 0 \quad (C) 5 \quad (D) 1 \quad (E) \text{None of the above}\)

13. For which of the following can we use the Squeeze Theorem to determine the limit of the function as \( x \) goes to zero?

\((A) f(x) = \frac{1}{x^2} \quad (B) g(x) = \ln(x - 1) \quad (C) h(x) = \cos(1/x) \quad (D) k(x) = \sqrt{x - 1} \quad (E) \text{None of the above}\)

14. Which of the following is(are) the vertical asymptotes of \( f(x) = \frac{9x^2 + 4}{(2x-1)^2} \)?

\((A) x = \frac{1}{2} \text{ and } x = \frac{9}{2} \quad (B) x = -\frac{1}{2} \text{ and } x = \frac{1}{2} \quad (C) x = -\frac{1}{2}, x = \frac{1}{2}, \text{ and } x = \frac{9}{2} \quad (D) x = \frac{9}{2} \quad (E) x = \frac{1}{2}\)
Calculus I: MAC2311
Spring 2019
Midterm 1 A
2/5/2019
Time Limit: 1 Hours 30 Minutes

Part II Instructions: 5 free response questions. Neatly give a complete solution to each problem and show all work and intermediate steps. We are grading the work and notation as well as the answer. Each problem is worth seven (7) points. A total of 35 points is possible on Part II. No credit will given without proper work. If we cannot read it and follow it, you will receive no credit for the problem.

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1. (7 pts) Find the derivative, $f'(a)$, for the function $f(x) = 2\sqrt{x-1}$ at a general point $x = a$, $a > 1$, using the limit definition for the derivative. (NOTE: NO credit will be given if another method is used.)

2. (6 pts) Let $f(x) = \begin{cases} x^2 - 1, & x < 0 \\ \frac{1}{x - 1}, & 0 < x < 3 \\ \frac{x^2 - 16}{x - 4}, & 3 \leq x \end{cases}$. Give all the values of $x$ at which each of the following types of discontinuities occur. If a type of discontinuity does not occur for the function, write NA in the correct space.

- Jump discontinuity:

- Removable discontinuity:

- Infinite discontinuity:
3. A particle is moving in a straight line with position \( s(t) = |2t - 4| \). The velocity of the particle is given by \( v(t) = \begin{cases} -2, & t < 2 \\ 2, & t > 2 \end{cases} \).

(a) (2 pts) Where is the velocity function continuous? Write your answer in interval notation.

(b) (4 pts) Does \( \lim_{t \to 2} v(t) \) exist? Justify your answer.

4. (8 pts) Find the following limit. Show all work. Do not use L’Hopital’s rule.
Infinite limits should be answered with “\( = \infty \)” or “\( = -\infty \)”, whichever is appropriate. If the limit does not exist (and cannot be answered as “\( \infty \)” or “\( -\infty \)”), state “DNE”.

\[
\lim_{x \to 1} \frac{\sqrt{10x - 9} - 1}{x - 1}
\]
5. (8 pts) Use the graph of \( f(x) \) below to evaluate the limits. Infinite limits should be answered with “= ∞” or “= −∞”, whichever is appropriate. If the limit does not exist (and cannot be answered as “∞” or “−∞”), state “DNE”.

\[
\begin{align*}
(a) \quad & \lim_{x \to -2^-} f(x) \\
(b) \quad & \lim_{x \to -2^+} f(x) \\
(c) \quad & \lim_{x \to 0} f(x) \\
(d) \quad & \lim_{x \to -2} f(x) \\
(e) \quad & \lim_{x \to 2} f(x) \\
(f) \quad & \lim_{x \to 4^-} f(x) \\
(g) \quad & \lim_{x \to 4} f(x) \\
(h) \quad & \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}
\end{align*}
\]