MAC 2312  
Spring 2018  

EXAM 2

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 2, 3.
   1 3 4 5 6 7 8 9 0
   1 2 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code”, encode C.
   A B D E

E. 1) This test consists of six 5-points, one 4-points multiple choice questions, two 1.5-
    points bonus multiple choice plus three free response questions worth 36 points for
    a total of 73 points. The test is counted out of 70 points.

   2) The time allowed is 60 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper, if you have a problem with your test
      or any emergency. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED
      WITH THE TEST.

F. KEEP YOUR SCANTRON COVERED AT ALL TIMES.

G. When you are finished:

   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in
      are there to stay.

   2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be
      prepared to show your UF ID card.

   3) Answers will be posted in E-Learning after the exam.

H. On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____________________________
DCT (Direct Comparison Test)  AST (Alternating Series Test)
LCT (Limit Comparison Test)  TFD (Test for Divergence—the nth term test)

Be sure to bubble your answers to questions 1–9 on your scantron.

#1 – #6 are worth 5 points each

1. Let \( \sum_{n=5}^{\infty} a_n = \sum_{n=5}^{\infty} (-1)^n \frac{3^n \cdot n!}{4 \cdot 9 \cdot 14 \cdots (5n-1)} \). Evaluate \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \).

A. \( \infty \)  B. \( \frac{3}{5} \)  C. \( \frac{5}{4} \)  D. \( \frac{4}{5} \)  E. 0

2. Which of the following series is/are convergent?

P. \( \sum_{n=9}^{\infty} \tan \left( \frac{1}{n} \right) \sqrt{n} \)  Q. \( \sum_{n=8}^{\infty} n \sin \left( \frac{1}{n^2} \right) \)  R. \( \sum_{n=4}^{\infty} \frac{\sin \left( \frac{1}{n} \right)}{n} \)

A. P and Q only  B. Q and R only  C. P and R only
D. P only  E. All of them

3. Which of the series below can be shown to be divergent using TFD?

A. \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \)
B. \( \sum_{n=1}^{\infty} \frac{n!}{n^n} \)
C. \( \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{7}} + \cdots \)
D. \( \sum_{n=1}^{\infty} \cos \left( \frac{1}{n^2} \right) \)
E. \( \sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n + 1} \)
4. Apply the root test to determine if each of the series below converges (C), diverges (D) or if the test is inconclusive (I).

P. \[ \sum_{n=7}^{\infty} \frac{4^n}{(2n + 1)3^{2n+1}} \]  
Q. \[ \sum_{n=5}^{\infty} \left( \frac{n}{n+1} \right)^n \]

A. P(C), Q(I)  
B. P(I), Q(C)  
C. P(C), Q(C)  
D. P(C), Q(D)  
E. P(D), Q(C)

5. Determine the limit of each sequence if it converges, or say 'div' if otherwise.

P. \( \left\{ \frac{(3n - 7)!}{(3n - 5)!} \right\} \)  
Q. \( \left\{ \frac{(-3)^n}{n} \right\} \)  
R. \( a_1 = \sqrt{2}, \ a_n = \sqrt{2 + a_{n-1}}, \) and \( \{a_n\} \) is monotonically increasing and bounded above

A. 0, 0, 0  
B. div, 0, 2  
C. div, div, 2  
D. div, div, \( \frac{\sqrt{2}}{2} \)  
E. 0, div 2

6. Find the sum of the series \( \sum_{n=2}^{\infty} \frac{(-3)^{n-1}}{2^n} \). (Note: starting index: \( n = 2 \))

A. \( -\frac{8}{63} \)  
B. divergent  
C. \( -\frac{3}{28} \)  
D. \( \frac{8}{99} \)  
E. \( \frac{8}{63} \)

#7 worth 4 points
7. Use Ratio test, determine if the series converges.

\[ 1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \cdots + \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n - 1)} + \cdots \]

A. The limit of the ratio test is e, hence the series diverges
B. The limit of the ratio test is \( \frac{1}{2} \), hence the series converges absolutely
C. The limit of the ratio test is \( \frac{1}{3} \), hence the series converges absolutely
D. The limit of the ratio test is \( \frac{1}{e} \), hence the series converges absolutely
E. The limit of the ratio test is \( \frac{\pi}{2} \), hence the series converges absolutely

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#8-#9, bonus worth 1.5 points each

8. Find the limit of the sequence, or 'DIV' if it diverges.

\[ \left\{ \frac{2}{3}, \frac{2}{3} + \frac{2}{9}, \frac{2}{3} + \frac{2}{9} + \frac{2}{27}, \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81}, \cdots \right\} \]

A. DIV  B. 3  C. \( \frac{4}{3} \)  D. \( \frac{3}{2} \)  E. 1

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9. According to the alternating series error estimation theorem, how many terms are required to estimate \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5n^3} \) with error being no bigger than \( \frac{1}{1000} \)?

A. 1  B. 2  C. 3  D. 4  E. 5

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End of Multiple Choice problems
1. Find the sum of the series \( \sum_{n=3}^{\infty} \left[ \sec \left( \frac{\pi}{n+2} \right) - \sec \left( \frac{\pi}{n} \right) \right] \).

(Note: index starts at \( n = 3 \))

(a) First, find a formula for the \( N^{th} \) partial sum \( S_N \). (show all your work)

\[ S_N = \]

(b) What is the limit of the \( N^{th} \) partial sum \( S_N \).

\[ \lim_{N \to \infty} S_N = \]

(c) Specify the sum of the series, or 'DIV' if it diverges.

\[ \text{Sum} = \]
2. (a) The series \( \sum_{n=3}^{\infty} \frac{1}{n(\ln n)^3} \) is convergent by the ________________ test.

(b) Determine if the series \( \sum_{n=3}^{\infty} \frac{\tan(\frac{1}{n})}{(\ln n)^3} \) converges by:

(i) First, state the statement of the test you will use:

(ii) Show your work determining if the series converges, be sure to show that the condition of the test you stated above is met in your work. (You may use the result from (a).)

The series is (convergent, divergent) (circle one) by the ________________ test.

3. (a) State the direct comparison test.

(b) State the alternating series test.
(c) Use the direct comparison test to show that the series \( \sum_{n=5}^{\infty} \frac{1}{\sqrt{n \ln n}} \) diverges.

(d) Use the alternating series test to show the series \( \sum_{n=5}^{\infty} \frac{(-1)^n}{\sqrt{n \ln n}} \) converges.

(e) Use (c) and (d) above to determine if the series \( \sum_{n=5}^{\infty} \frac{(-1)^n}{\sqrt{n \ln n}} \) is (absolute convergent, conditional convergent, diverges). (circle one)

(f) Use the sum of the first 4 terms to approximate the sum of the series \( \sum_{n=5}^{\infty} \frac{(-1)^n}{\sqrt{n \ln n}} \).
   (no need to combine the terms).

\[ \text{Sum} \approx \]

(g) Estimate the error involved in this estimation.

(No need to simplify).

\[ |\text{error}| \leq \]

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