Name: _______________________________  Section #: __________________

UF-ID: _______________________________  TA Name: _________________

A: Sign the back of your scantron sheet **in ink**.

B: On the indicated spaces in front of your scantron, **in pencil**, write and encode:
   1. Your name (last name, first initial, middle initial)
   2. Your UF-ID number
   3. Your 4-digit section number

C: Under “special codes”, code in the test ID number 4,1.
   1  2  3 •  5  6  7  8  9  0
   •  2  3  4  5  6  7  8  9  0

D: At the top right of your answer sheet, for “Test Form Code”, encode A.
   •  B  C  D  E

E: Some basic information about the exam:
   1. This exam has 22-question multiple choice questions each worth 3 points. The entire exam is worth 66/60 points.
   2. You will have 2 hours to take the exam.
   3. You may write on your exam.
   4. Raise your hand if you need more scratch paper or if you have a problem with the test. **DO NOT LEAVE YOUR SEAT UNLESS YOUR ARE FINISHED WITH THE EXAM.**

F: **KEEP YOUR SCANTRON COVERED AT ALL TIMES**

G: When you are finished:
   1. Before turning in your exam, check for transcribing errors. Any mistakes you leave there are there to stay.
   2. Turn in your scantron to your TA, or the proctor designated as your TA. Be prepared to show your UF-ID card.
   3. Solutions to the exam will be posted on Canvas after the exam is over

The **Honor Pledge**: “On my honor, I have neither given or received unauthorized aid doing this exam.”

Signature: ___________________________________________________________________
List of Common Maclaurin Series

1. \[
\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ I.O.C. is } (-1,1)
\]

2. \[
\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}, \text{ I.O.C. is } [-1,1)
\]

3. \[
\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}, \text{ I.O.C. is } (-1,1]
\]

4. \[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ I.O.C. is } (-\infty,\infty)
\]

5. \[
\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ I.O.C. is } (-\infty,\infty)
\]

6. \[
\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ I.O.C. is } (-\infty,\infty)
\]

7. \[
arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ I.O.C. is } [-1,1]
\]
There are 22 questions on the exam. Fill in the answers to these questions on the provided scantron sheet. Only answers on the scantron will be graded. Each problem is worth 3 points, for a total of 66 points on this portion of the exam.

1. For what values of $d$ does the series \( \sum_{n=1}^{\infty} \frac{n^d}{1 + \sqrt{n}} \) converge?
   
   (a) \( d \in (-1, 1) \)
   
   (b) \( d \in (-\infty, -\frac{1}{2}] \)
   
   (c) \( d \in [\frac{1}{2}, \infty) \)
   
   (d) \( d \in (-\infty, -\frac{1}{2}) \)
   
   (e) \( d \in (\frac{1}{2}, \infty) \)

2. Compute \( \int_0^2 x^2 e^{x/2} \, dx \).
   
   (a) \( 40e - 16 \)  
   
   (b) \( 8e - 16 \)  
   
   (c) \( \frac{16e}{3} \)  
   
   (d) \( \frac{5e - 1}{4} \)  
   
   (e) \( \frac{4e}{3} \)

3. What sort of tangent line does the parametric curve \( x = \sin(\pi t) \), \( y = t^2 - t - 3 \), \( 0 \leq t \leq 3 \) have when \( t = \frac{3}{2} \)?
   
   (a) A vertical tangent line
   
   (b) A horizontal tangent line
   
   (c) A tangent line with slope 2
   
   (d) A tangent line with slope \( \frac{9}{4} \)
   
   (e) There is no tangent line for this value of $t$
4. Consider the solid of revolution formed by revolving the area bounded by the curve $y = \frac{1}{x}$, the line $x = 1$, the line $y = 0$, and the line $x = a$, $(a > 1)$ about the $x$-axis (picture shown below). What is an integral representing the volume of this solid?

(a) $\pi \int_1^a \frac{dx}{x}$  
(b) $2\pi \int_1^a \frac{dx}{x}$  
(c) $\pi \int_1^a \frac{dx}{x^2}$  
(d) $2\pi \int_1^a \frac{dx}{x^2}$  
(e) $\pi \int_1^a \frac{dx}{\sqrt{x}}$

5. Which of the following is the correct form of the partial fraction decomposition of $\frac{2x^4 + 16}{(x^2 + 1)x^2}$?

(a) $2 + \frac{Ax + B}{x^2 + 1} + \frac{C}{x^2}$
(b) $\frac{Ax + B}{x^2 + 1} + \frac{C}{x} + \frac{D}{x^2}$
(c) $\frac{A}{x^2 + 1} + \frac{B}{x^2} + \frac{C}{x}$
(d) $\frac{Ax + B}{x^2 + 1} + \frac{C}{x^2}$
(e) $2 + \frac{Ax + B}{x^2 + 1} + \frac{C}{x} + \frac{D}{x^2}$

6. Evaluate $\int_1^e (\ln(x))^2 \, dx$

(a) $e - \frac{e^2 + 1}{2}$  
(b) $e - \frac{e^2}{2}$  
(c) $\frac{1}{3}$  
(d) $\frac{e^3}{3}$  
(e) $e - 2$
7. The graph of a polar function over the range $0 \leq \theta < \infty$ is given above. Which of the following is the function being graphed?

(a) $r^2 = 3 \cos \theta$
(b) $r = 3 + \theta$
(c) $r = \frac{3}{1 + \theta}$
(d) $r = 3 + \sin \theta$
(e) $r = 3 \cos(8\theta)$

8. Evaluate the improper integral

$$\int_{3}^{\infty} \frac{dx}{x \ln(x) \ln(\ln(x))}.$$ 

(a) $\frac{1}{\ln(\ln(\ln(3)))}$
(b) $\ln(\ln(\ln(3)))$
(c) $-\ln(\ln(\ln(3)))$
(d) Diverges

9. The base of a solid is the triangle in the $xy$-plane with vertices (0, 0), (1, 0), and (0, 1). The cross-sections of the solid perpendicular to the $x$-axis are squares. What is the volume of the solid?

(a) $\frac{1}{3}$
(b) $\frac{2}{3}$
(c) 3
(d) $3\pi$
10. Which of these integrals would compute the area of the region that is inside the circle \( r = 4 \cos \theta \) but outside the circle \( x^2 + y^2 = 4 \)?

(a) \[ \int_{0}^{\pi/3} 16 - 16 \cos^2 \theta \, d\theta \]

(b) \[ \int_{0}^{\pi/3} 16 \cos^2 \theta - 4 \, d\theta \]

(c) \[ \pi \int_{-\pi/3}^{\pi/3} 4 - 16 \cos^2 \theta \, d\theta \]

(d) \[ \frac{1}{2} \int_{-\pi/3}^{\pi/3} 4 \cos \theta - 2 \, d\theta \]

(e) \[ \int_{-\pi/3}^{\pi/3} 16 \cos^2 \theta - 16 \, d\theta \]

11. Suppose that a function \( y = f(x) \) is given with \( f(x) \geq 0 \) for \( 0 \leq x \leq 4 \). If the area under the graph of \( f(x) \) from \( x = 0 \) to \( x = 4 \) is revolved about the \( x \)-axis, then the volume of the solid of revolution is given by

(a) \[ 2\pi \int_{0}^{4} x[f(x)]^2 \, dx \]

(b) \[ \pi \int_{0}^{4} x^2f(x) \, dx \]

(c) \[ 2\pi \int_{0}^{4} \sqrt{1 + [f(x)]^2} \, dx \]

(d) \[ \pi \int_{0}^{4} [f(x)]^2 \, dx \]

(e) \[ 2\pi \int_{0}^{4} [f(x)]^2 \, dx \]

12. The integral

\[ \int \frac{1}{(x^2 + 1)^2} \, dx = A \arctan(x) + B \left( \frac{x}{x^2 + 1} \right) + C. \]

Find \( A + B \).

(a) 0 \hspace{1cm} (b) 1 \hspace{1cm} (c) \frac{3}{4} \hspace{1cm} (d) 2
13. Use a known Maclaurin series to evaluate the sum of the series $\frac{1}{4} - \frac{1}{2 \cdot 4^2} + \frac{1}{3 \cdot 4^3} - \frac{1}{4 \cdot 4^4} + \cdots$.

(a) $(n - 1)!(1 - e^{-1/4})$
(b) $\ln \left( \frac{5}{4} \right)$
(c) $\frac{1}{8}$
(d) $\ln \left( \frac{1}{4} \right)$
(e) $\ln \left( \frac{1}{4} \right) \frac{1}{4}$

14. Suppose that $F(x)$ is a power series with radius of convergence $R = 4$. Which below is false?

(a) $F(3x)$ is a power series with $R = \frac{4}{3}$.
(b) $F'(x)$ is a power series with $R = 4$.
(c) $\int F(x) \, dx$ is a power series with $R = 4$.
(d) $F \left( \frac{x}{4} \right)$ is a power series with $R = 16$.
(e) $xF(3x)$ is a power series with $R = 4$.

15. Which of the following best describes the graph of the parametric equation $x(t) = 3 \sin t, y(t) = 4 \cos t$?

(a) A circle that is traced out clockwise.
(b) An ellipse that is traced out clockwise.
(c) A circle that is traced out counterclockwise.
(d) An ellipse that is traced out counterclockwise.
(e) A hyperbola traced from left to right.
16. Which of the following integrals would you get using the **shell method** to find the volume of the solid formed by revolving the region bounded by the graphs of \( y = x^3 \), \( y = 8 \) and \( x = 1 \) about the line \( x = 2 \)? (Region shown below)

(a) \( 2\pi \int_1^8 (2 - y)(1 - \sqrt[3]{y}) \, dy \)

(b) \( \pi \int_1^2 \left[ 8^2 - (x^3)^2 \right] \, dx \)

(c) \( 2\pi \int_1^8 (\sqrt[3]{y} - 1)(8 - y) \, dy \)

(d) \( 2\pi \int_1^2 (2 - x)(8 - x^3) \, dx \)

(e) \( \pi \int_1^8 (2 - y)(1 - \sqrt[3]{y}) \, dy \)

17. Graph the polar curves \( r = 2 + 2\sin \theta \) and \( r = 4\sin \theta \). At how many distinct points do the graphs intersect?

(a) 0  (b) 1  (c) 2  (d) 3  (e) 4
18. Use the second degree Taylor Polynomial, $T_2$, for $f(x) = \sin x$ centered at $\frac{\pi}{4}$ to approximate $\sin\left(\frac{\pi}{5}\right)$.

(a) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{80}$

(b) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{40} + \frac{\sqrt{2}\pi^2}{80}$

(c) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{1600}$

(d) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{40} - \frac{\sqrt{2}\pi^2}{1600}$

(e) $\sin\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{40} + \frac{\sqrt{2}\pi^2}{80}$

19. For what range of values of $t$ will the graph of the parametric curve $x = \frac{t^2}{2} + t, y = \frac{t^3}{3} - t$ have negative slope and be concave down?

(a) $t < -1$  (b) $t > -1$  (c) $-1 < t < 1$  (d) $t < 1$  (e) $t > 1$

20. Consider the power series $S = \sum_{n=1}^{\infty} c_n(x-5)^n$. Which of the following could be an interval of convergence for $S$?

(a) $IOC = [-1, 7)$

(b) $IOC = [-4, -2]$

(c) $IOC = [-5, 15)$

(d) $IOC = (4, 7]$

(e) $IOC = [3, 6)$
21. Evaluate \( \int \frac{e^{2x}}{(e^{2x} + 1)(e^x + 1)} \, dx \).

(a) \( \ln(e^{2x} + 1) + \frac{1}{2} \arctan(e^x) - \frac{1}{2} \ln |e^x + 1| + C \)

(b) \( \frac{1}{2} \ln(e^{2x} + 1) + \frac{1}{2} \arctan(e^x) - \frac{1}{2} \ln |e^x + 1| + C \)

(c) \( -\frac{1}{2} \ln(e^{2x} + 1) + \frac{1}{2} \arctan(e^x) + \frac{1}{2} \ln |e^x + 1| + C \)

(d) \( -\ln(e^{2x} + 1) + \frac{1}{2} \arctan(e^x) + \frac{1}{2} \ln |e^x + 1| + C \)

(e) \( \frac{1}{4} \ln(e^{2x} + 1) + \frac{1}{2} \arctan(e^x) - \frac{1}{2} \ln |e^x + 1| + C \)

22. Which of the following statements about convergence is true?

(a) If \( a_n \) and \( b_n \) are two positive sequences with \( a_n \leq b_n \) and \( \sum_{n=0}^{\infty} b_n \) diverges, then \( \sum_{n=0}^{\infty} a_n \) diverges by the Direct Comparison Test.

(b) If \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = 1 \), then \( \sum_{n=0}^{\infty} a_n \) diverges by the Root Test.

(c) If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \), then \( \sum_{n=0}^{\infty} a_n \) converges by the Ratio Test.

(d) If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=0}^{\infty} a_n \) converges by the Test for Divergence.

(e) If \( a_n \) and \( b_n \) are two positive sequences with \( \lim_{n \to \infty} \frac{a_n}{b_n} = 1 \) and \( \sum_{n=0}^{\infty} b_n \) diverges, then \( \sum_{n=0}^{\infty} a_n \) diverges by the Limit Comparison Test.