A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode on your scantron in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 2, 2.
   \[1 \bullet 3 4 5 6 7 8 9 0\]
   \[1 \bullet 3 4 5 6 7 8 9 0\]

D. At the top right of your answer sheet, for “Test Form Code”, encode B.
   \[A \bullet C D E\]

E. 1) There are seven 2-point multiple choice questions, seven 3-point multiple choice questions, one 2-point bonus multiple choice questions, plus four free response questions for a total of 62 points.
   2) The time allowed is 90 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR SCANTRON COVERED AT ALL TIMES.

G. When you are finished:
   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
   2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.
   3) Answers will be posted in E-Learning after the exam.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid in doing this exam."

Student’s Signature:__________________
Questions 1–7 are worth 2 point each.

1. Determine the convergence(C) or divergence(D) of each series using any method.

1. \[ \sum_{n=5}^{\infty} \frac{\sqrt{2n^2 + 1}}{n^3} \]

2. \[ \sum_{n=5}^{\infty} \frac{1}{(\ln n)^6} \]

3. \[ \sum_{n=5}^{\infty} \sin \left( \frac{1}{n^5} \right) \]

A. 1C, 2C, 3D  
B. 1C, 2D, 3C  
C. 1C, 2C, 3C  
D. 1C, 2C, 3D  
E. 1D, 2D, 3C

2. Determine which statement(s) is/are **TRUE**.

1. If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=3}^{\infty} a_n \) diverges.

2. The series \( \sum_{n=1}^{\infty} \frac{1}{n \ln n} \) is convergent.

3. If \( 0 < a_n < b_n \) and \( \sum_{n=1}^{\infty} b_n \) diverges, then \( \sum_{n=1}^{\infty} a_n \) diverges.

4. Suppose both \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) are divergent. Then \( \sum_{n=1}^{\infty} (a_n - b_n) \) converges.

5. Both \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \) and \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n} \) are convergent.

A. 2, 4 and 5 only  
B. 1 only  
C. 1 and 2 only  
D. 2 and 3 only  
E. 2, 5 only
3. Apply the **Ratio test** to determine which series are convergent (C) or divergent (D), or inconclusive (I).

1. \[ \sum_{n=3}^{\infty} \frac{n^2}{n^8 + 1} \]

2. \[ \sum_{n=5}^{\infty} \frac{1}{\ln n} \]

3. \[ \sum_{n=8}^{\infty} \frac{n^{n+1}}{n!} \]

A. 1I, 2D, 3C

B. 1I, 2I, 3D

C. 1C, 2I, 3D

D. 1C, 2D, 3D

E. 1I, 2I, 3C

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4. Which infinite sequence converge(s)?

1. \{\ln(9n + 2) - \ln(-9 + 3n)\}

2. \(\left\{ \frac{(-1)^n}{\sqrt{n}} \right\}\)

3. \(\left\{ \cos \left( \frac{\pi}{n} \right) \right\}\)

A. 1, 2, 3

B. 3 only

C. 1, 2 only

D. 1, 3 only

E. 2, 3 only

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5. Approximate the sum of the series \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)} \] using the first 3 terms. Find the least upper estimate to the error using this approximation. (\(|R_3| < \) ___)

A. \(\frac{1}{2}\)

B. \(\frac{1}{72}\)

C. 1

D. \(\frac{1}{20}\)

E. \(\frac{1}{5}\)
6. Which of the following series converge?

I. \[ \sum_{n=5}^{\infty} \frac{\tan\left(\frac{1}{\sqrt{n}}\right)}{n} \]

II. \[ \sum_{n=5}^{\infty} n \sin\left(\frac{1}{n^2}\right) \]

III. \[ \sum_{n=5}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right) \]

IV. \[ \sum_{n=5}^{\infty} \cos\left(\frac{1}{n^3}\right) \]

A. I and III only  
B. I, II and III only  
C. I only  
D. none  
E. all

7. Determine whether the series converges absolutely (A), conditionally (C) or diverges (D).

1. \[ \sum_{n=3}^{\infty} (-1)^n \cos\left(\frac{1}{n^5}\right) \]

2. \[ \sum_{n=5}^{\infty} \frac{\cos(n\pi)}{n^2} \]

3. \[ \sum_{n=8}^{\infty} \frac{3 - (-1)^n}{n} \]

4. \[ \sum_{n=5}^{\infty} \frac{(-1)^n}{n^{1/2} \ln(n)} \]

A. 1D, 2A, 3D, 4A  
B. 1C, 2D, 3D, 4C  
C. 1D, 2A, 3D, 4C  
D. 1C, 2A, 3C, 4C  
E. 1D, 2C, 3C, 4C

Questions 8 - 14 are worth 3 points each.

8. Find the sum of the series \[ \sum_{n=5}^{\infty} \frac{2}{n^2 - 1} \].

A. \[ \frac{1}{3} \]  
B. \[ \frac{9}{20} \]  
C. \[ \frac{1}{2} \]  
D. \[ \frac{1}{4} \]  
E. 1
9. Select the most accurate statement about the series.

\[ \sum_{n=5}^{\infty} \arctan n - \arctan(n + 1) \]

A. The series converges absolutely by the direct comparison test
B. The series converges by the telescoping series test
C. The series diverges by the telescoping series test
D. The series converges by the alternating series test
E. The series diverges by the test for divergence

10. Determine the values of \( k \) for which the series \( \sum_{n=1}^{\infty} \frac{n^{1/2}}{\sqrt[3]{n^k + 10n}} \) will converge. (express the values in interval notation)

A. \((5, \infty)\)  \hspace{1cm} B. \((\frac{9}{2}, \infty)\)  \hspace{1cm} C. \((\frac{3}{2}, \infty)\)  \hspace{1cm} D. \([\frac{9}{2}, \infty)\)  \hspace{1cm} E. \((\frac{9}{2}, \frac{3}{2})\)

11. Let \( \{a_n\} \) be a sequence defined as \( a_1 = 0 \) and \( a_{n+1} = \frac{a_n + 3}{3a_n + 10} \). It is known that \( \{a_n\} \) is monotonically increasing and \( 0 \leq a_n < 1 \). Find the limit of the sequence if it exists.

A. \(\frac{3 - \sqrt{5}}{2}\)  \hspace{1cm} B. \(\frac{-3 + \sqrt{5}}{2}\)  \hspace{1cm} C. \(\frac{-3 + \sqrt{13}}{2}\)
D. \(\frac{3 + \sqrt{13}}{2}\)  \hspace{1cm} E. limit does not exist

12. Find the sum of the series \( \sum_{n=1}^{\infty} \sin^n \left(\frac{7\pi}{6}\right) \).

A. \(-\frac{3}{4}\)  \hspace{1cm} B. \(\frac{1}{6}\)  \hspace{1cm} C. \(-\frac{9}{20}\)  \hspace{1cm} D. \(-\frac{1}{3}\)  \hspace{1cm} E. \(\frac{7}{20}\)
13. Assume that \( \left| \frac{a_{n+1}}{a_n} \right| \) converges to \( \rho = \frac{1}{8} \). Apply the **Ratio test**, what can you say about the convergence of the series \( \sum_{n=5}^{\infty} n^7 a_n \)?

A. the test is inconclusive
B. the series diverges
C. the series converges

14. Let \( S = \sum_{n=1}^{\infty} a_n \) such that \( S_N = 4 - \frac{4}{N^2} \). Find \( a_5 = \) _____.

A. \( \frac{9}{100} \)  
B. \( \frac{433}{2700} \)  
C. \( \frac{13}{1075} \)  
D. \( \frac{13}{107} \)  
E. \( \frac{19}{2025} \)

**Bonus Question 15 is worth 2 points.**

15. Determine which statement is **true** using the root test.

\[ \sum_{n=1}^{\infty} \left( 1 - \frac{1}{n} \right)^n \]

A. The root test is inconclusive
B. The series converges absolutely by the root test
C. The series diverges by the root test

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**End of Multiple Choice problems**
YOU MUST SHOW ALL WORK TO RECEIVE FULL CREDIT.

Specify the test(s) you use, clearly show the statement(s) of the test(s) in your work, and be sure to show the condition(s) of the test(s) is/are met in your work.

1. (7 points) Determine if the series converges.

\[
1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \ldots + \frac{n!}{1 \cdot 3 \cdot 5 \cdot (2n-1)} + \ldots
\]
2. (4 pts) Determine the values of $x$ for which the series converges. What test do you use?

$$\sum_{n=4}^{\infty} \frac{(\sin x)^{2n}}{3n^2}$$

(write your answer in interval notation at the end of the question)

$x \in \underline{\text{____________}}$ by using $\underline{\text{____________}}$ test.

3. (4 pts) Determine the convergence of the series. What test do you use?

$$\sum_{n=4}^{\infty} \frac{\cos \left( \frac{1}{n} \right)}{n^{1/n}}$$
4. (10 points) Determine if the series converges and find the exact sum if it does.

\[ \sum_{n=3}^{\infty} \ln \left( 1 - \frac{1}{n^2} \right). \]