A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion Section number

C. Under “special codes”, code in the test number 3, 1.

   1  2  •  4  5  6  7  8  9  0
   •  2  3  4  5  6  7  8  9  0

D. At the top right of your answer sheet, for “Test Form Code” encode A.

   •  B  C  D  E

E. This test consists of 10 five-point, one three-point and two one-point multiple choice questions, bonus questions worth 4 points and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or proctor. Be prepared to show your picture ID with a legible signature.

3) The answers will be posted on the MAC 2311 homepage after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted on Elearning within one week.
NOTE: Be sure to bubble the answers to questions 1–17 on your scantron.

Questions 1 - 10 are worth 5 points each.

1. Find the absolute maximum and minimum values of \( f(x) = e^{x^3-3x+2} \) on \([0, 2]\).
   a. \( e^4, 1 \)    b. \( e^2, 1 \)    c. \( e^4, 0 \)    d. \( e^2, e \)    e. \( e^4, e^2 \)

2. Find each value of \( x \) at which \( f(x) = x^2 + \ln(x^2) \) has an inflection point.
   a. \( x = 0, x = \pm 1 \)    b. \( x = 0, x = \pm 2 \)    c. \( x = \pm 2 \) only
   d. \( x = \pm 1 \) only    e. \( f \) has no inflection points

3. A particle moves along the graph of \( y = \tan(x) \sec(x) \) so that \( y \) is increasing at a rate of 3 units per second. Find the rate at which \( x \) is changing with respect to time when \( x = \frac{\pi}{4} \).
   a. \( x \) is increasing by \( \frac{3}{\sqrt{2}} \) units per second.
   b. \( x \) is decreasing by \( \sqrt{2} \) units per second.
   c. \( x \) is increasing by \( \frac{1}{3\sqrt{2}} \) units per second.
   d. \( x \) is decreasing by \( \frac{\sqrt{2}}{3} \) units per second.
   e. \( x \) is increasing by \( \frac{1}{\sqrt{2}} \) units per second.
4. Find all values of \( c \) guaranteed by the Mean Value Theorem for 
\[ f(x) = x^3 - 4x \] on \([0, 3]\).

a. \( c = \sqrt{3} \) only 

b. \( c = -\frac{1}{\sqrt{3}} \) and \( c = \frac{1}{\sqrt{3}} \) 

c. \( c = 1 \) only  

d. \( c = -\sqrt{3} \) and \( c = \sqrt{3} \) 

e. \( c = \frac{1}{\sqrt{3}} \) only  

5. Find the critical numbers of 
\[ f(x) = \sin^2 x - \cos x \] on \([0, 2\pi]\).

a. \( x = \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3} \) 

d. \( x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6} \)  

e. \( x = \frac{2\pi}{3}, \frac{4\pi}{3} \)  

b. \( x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \) 

c. \( x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \) 

6. A particle moves in a straight line so that its position (in feet from a starting point) after \( t \) minutes is given by 
\[ s(t) = 6t^2 - t^3. \]  
Find the displacement of the particle and the total distance traveled on the time interval \( 0 \leq t \leq 5 \).

<table>
<thead>
<tr>
<th>displacement</th>
<th>total distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 32 ft</td>
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<tr>
<td>b. 25 ft</td>
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<td>57 ft</td>
</tr>
<tr>
<td>e. 32 ft</td>
<td>39 ft</td>
</tr>
</tbody>
</table>
7. The local extrema of \( f(x) = \frac{(x - 4)^2}{x} \) occur at which of the following \( x \)-values?

<table>
<thead>
<tr>
<th>local minimum</th>
<th>local maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>at ( x = )</td>
<td>at ( x = )</td>
</tr>
<tr>
<td>a. 4</td>
<td>none</td>
</tr>
<tr>
<td>b. 0</td>
<td>-4, 4</td>
</tr>
<tr>
<td>c. 4</td>
<td>-4</td>
</tr>
<tr>
<td>d. -4, 4</td>
<td>0</td>
</tr>
<tr>
<td>e. -4</td>
<td>4</td>
</tr>
</tbody>
</table>

8. The radius of a sphere is measured to be \( r = 4 \) inches. If the measurement of \( r \) has a possible error of \( \pm 0.04 \) inch, use differentials to approximate the percentage error that can occur in using this measure to compute the surface area of the sphere.

Note: the surface area \( S \) of a sphere is given by \( S = 4\pi r^2 \).

a. 1%  b. 2.4%  c. 1.02%  d. 2.01%  e. 2%

9. If \( f(x) = \frac{x^2 - 3}{x^3} \), then \( f'(x) = \frac{9 - x^2}{x^4} \) and \( f''(x) = \frac{2x^2 - 36}{x^5} \).

Find each interval on which \( f \) is both increasing and concave up.

a. \((-3, 0) \cup (3\sqrt{2}, \infty)\)  b. \((-3\sqrt{2}, -3)\)  c. \((-3, 3)\)

d. \((-3, 0)\) only  e. \((-\infty, -3\sqrt{2}) \cup (3, \infty)\)
10. Which of the following statements is/are true of \( f(x) = (x^2 - 1)^{1/3} \)?

Note that \( f'(x) = \frac{2x}{3(x^2 - 1)^{2/3}} \).

A. \( f(x) \) has exactly one critical number.
B. \( f(x) \) has vertical tangent lines at \( x = -1 \) and \( x = 1 \).
C. \( f(x) \) is increasing on \((0,1)\) and \((1,\infty)\).

a. A only b. B and C only c. A and C only
d. B only e. A, B, and C

11. (3 points) The position function of a snail moving along a straight path is \( s(t) = \frac{t - 3}{t + 2} \), where \( s(t) \) is the distance in centimeters from the starting point after \( t \) minutes. Find the acceleration of the snail after 1 minute.

a. \( \frac{5}{9} \) cm/min\(^2\)  b. \(-1\) cm/min\(^2\)  c. \(-\frac{10}{27}\) cm/min\(^2\)
d. \( \frac{1}{9} \) cm/min\(^2\)  e. \(-\frac{2}{27}\) cm/min\(^2\)

The following true/false questions are worth one point each.

12. The position function for an object moving in a straight line is given by \( s = f(t) \). If \( f'(b) > 0 \) and \( f''(b) < 0 \), then the object is slowing down at time \( t = b \).

a. True b. False
13. If \( y = f(x) \) is a continuous function so that \( f'(3) = 0 \) and \( f''(3) = -4 \), then \( f \) has a relative minimum at \( x = 3 \).

a. True \hspace{1cm} b. False

**Bonus!! (one point each)** The graph of the derivative of a continuous function \( f \) is sketched below.

![Graph of the derivative](image)

Use the graph to determine if the following statements about \( f \) are true or false.

14. \( f(x) \) has a local minimum at \( x = 0 \).

a. True \hspace{1cm} b. False

15. \( f \) is increasing on the interval \((-1, 1)\) only.

a. True \hspace{1cm} b. False

16. \( f \) is concave down on intervals \((-\infty, -1)\) and \((1, \infty)\).

a. True \hspace{1cm} b. False

17. \( f \) has inflection points at \( x = -1 \) and \( x = 1 \).

a. True \hspace{1cm} b. False
1. Let $f(x) = x^{\cos x}$

   a. Use logarithmic differentiation to find $f'(x)$.

   $f'(x) =$

   b. Find the slope of the tangent line to $f(x)$ at $x = \pi$.

   $m =$
2. To which of the functions below can we apply Rolle’s Theorem on the interval [0, 8]? Write “can apply” or “cannot apply”. If you can apply Rolle’s Theorem, find each value of c guaranteed by the theorem. If not, state a condition not satisfied by the function.

a. \( f(x) = \frac{x^2 - 8x}{x - 1} \)

b. \( f(x) = x^{2/3} - 2x^{1/3} \)

3. A boy standing at the end of a pier which is 12 feet above the water is pulling on a rope that is attached to a rowboat in the water. If he is pulling the rope in at 6 feet per minute, how fast is the boat docking (moving towards the pier) when it is 16 feet from the pier?

The boat is docking at __________ feet per minute.
4. Let $f(x) = \sqrt{1 + 2x}$.

A. Find the linearization $L(x)$ of $f$ at $a = 4$.

$$L(x) = \frac{1}{2 \sqrt{1 + 2a}}$$

B. Use $L(x)$ to approximate $\sqrt{9.06}$. Hint: $\sqrt{9.06} = f(4.03)$.

Write your answer as a single fraction or a decimal.
5. Let \( f(x) = 4x^5 - 5x^4 \).

A. Determine the following for \( f \) (if none, write “none”):

(a) \( x \)-intercept(s) ______

(b) Show the signs of \( f' \) and \( f'' \) on the number lines below:

\[
\begin{array}{c}
\text{-----} \\
\text{_____________} \ f' \ 
\text{______________} \ f'' \\
\end{array}
\]

(c) local maximum at \( x = \)______  local maximum value: ______

(d) local minimum at \( x = \)______  local minimum value: ______

(e) inflection point(s) at \( x = \)______

B. Sketch the graph of \( y = 4x^5 - 5x^4 \), using the information above. Label all important features of the graph. Note that \( f \left( \frac{3}{4} \right) \approx -0.6 \).
MAC 2311   Spring 2010
Exam 3B

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
3) Discussion Section number

C. Under "special codes", code in the test number 3, 2.

1 2   4 5 6 7 8 9 0
1 3   4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode B.
A   C   D   E

E. This test consists of 10 five-point, one three-point and two one-point multiple choice questions, bonus questions worth 4 points and two sheets (4 pages) of partial credit questions worth 25 points. The time allowed is 90 minutes.

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3) The answers will be posted on the MAC 2311 homepage after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted on Elearning within one week.
1. Find each value of \( x \) at which \( f(x) = x^2 + \ln(x^2) \) has an inflection point.
   a. \( x = 0, x = \pm 1 \)  
   b. \( x = \pm 1 \) only  
   c. \( x = 0, x = \pm 2 \)  
   d. \( x = \pm 2 \) only  
   e. \( f \) has no inflection points.

2. Find the absolute maximum and minimum values of \( f(x) = e^{x^3-3x+2} \) on \([0, 2]\).
   a. \( e^2, e \)  
   b. \( e^4, e^2 \)  
   c. \( e^4, 1 \)  
   d. \( e^4, 0 \)  
   e. \( e^2, 1 \)

3. A particle moves in a straight line so that its position (in feet from a starting point) after \( t \) minutes is given by \( s(t) = 6t^2 - t^3 \). Find the displacement of the particle and the total distance traveled on the time interval \( 0 \leq t \leq 5 \).

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4. The radius of a sphere is measured to be $r = 3$ inches. If the measurement of $r$ has a possible error of $\pm 0.03$ inch, use differentials to approximate the percentage error that can occur in using this measurement to compute the surface area of the sphere.

Note: the surface area $S$ of a sphere is given by $S = 4\pi r^2$.

a. 2.4%  
   b. 1%  
   c. 2.01%  
   d. 1.02%  
   e. 2%

5. A particle moves along the graph of $y = \tan(x) \sec(x)$ so that $y$ is increasing at a rate of 3 units per second. Find the rate at which $x$ is changing with respect to time when $x = \frac{\pi}{4}$.

   a. $x$ is increasing by $\frac{1}{\sqrt{2}}$ units per second.
   
   b. $x$ is increasing by $\frac{1}{3\sqrt{2}}$ units per second.
   
   c. $x$ is decreasing by $\frac{\sqrt{2}}{3}$ units per second.
   
   d. $x$ is increasing by $\frac{3}{\sqrt{2}}$ units per second.
   
   e. $x$ is decreasing by $\sqrt{2}$ units per second.

6. Find all values of $c$ guaranteed by the Mean Value Theorem for $f(x) = x^3 - 4x$ on $[0, 3]$.

   a. $c = 1$ only  
   b. $c = -\sqrt{3}$ and $c = \sqrt{3}$  
   c. $c = \frac{1}{\sqrt{3}}$ only
   
   d. $c = \sqrt{3}$ only  
   e. $c = -\frac{1}{\sqrt{3}}$ and $c = \frac{1}{\sqrt{3}}$
7. Find the critical numbers of \( f(x) = \sin^2 x - \cos x \) on \([0, 2\pi]\):

a. \( x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6} \)
b. \( x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \)
c. \( x = \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3} \)
d. \( x = \frac{2\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3} \)
e. \( x = 0, \pi, \frac{5\pi}{3} \)

8. The local extrema of \( f(x) = \frac{(x - 1)^2}{x} \) occur at which of the following \( x \)-values?

local minimum
at \( x = \)________

local maximum
at \( x = \)_______

a. 0
b. -1
b. 1
d. 1
e. -1, 1

9. Which of the following statements is/are true of \( f(x) = (x^2 - 4)^{1/3} \)?

Note that \( f'(x) = \frac{2x}{3(x^2 - 4)^{2/3}} \).

A. \( f(x) \) has exactly one critical number.
B. \( f(x) \) is increasing on \((0, 2)\) and \((2, \infty)\).
C. \( f(x) \) has vertical tangent lines at \( x = -2 \) and \( x = 2 \).

da. B and C only
b. C only
c. A, B and C
d. A only
e. A and B only
10. If \( f(x) = \frac{x^2 - 3}{x^3} \), then \( f'(x) = \frac{9 - x^2}{x^4} \) and \( f''(x) = \frac{2x^2 - 36}{x^5} \).

Find each interval on which \( f \) is both increasing and concave up.

a. \((-3, 3)\) \hspace{1cm} b. \((-\infty, -3\sqrt{2}) \cup (3, \infty)\) \hspace{1cm} c. \((-3\sqrt{2}, -3)\)

d. \((-3, 0) \cup (3\sqrt{2}, \infty)\) \hspace{1cm} e. \((-3, 0)\) only

11. (3 points) The position function of a snail moving along a straight path is \( s(t) = \frac{t - 3}{t + 2} \), where \( s(t) \) is the distance in centimeters from the starting point after \( t \) minutes. Find the acceleration of the snail after 1 minute.

a. \( \frac{1}{9} \) cm/min\(^2\) \hspace{1cm} b. \( -\frac{2}{27} \) cm/min\(^2\) \hspace{1cm} c. \( \frac{5}{9} \) cm/min\(^2\)

d. \( -\frac{10}{27} \) cm/min\(^2\) \hspace{1cm} e. \( -1 \) cm/min\(^2\)

The following true/false questions are worth one point each.

12. If \( y = f(x) \) is a continuous function so that \( f'(6) = 0 \) and \( f''(6) = 2 \), then \( f \) has a relative maximum at \( x = 3 \).

a. True \hspace{1cm} b. False

13. The position function for an object moving in a straight line is given by \( s = f(t) \). If \( f'(b) > 0 \) and \( f''(b) < 0 \), then the object is slowing down at time \( t = b \).

a. True \hspace{1cm} b. False
Bonus!! (one point each) The graph of the derivative of a continuous function $f$ is sketched below.

Use the graph to determine if the following statements about $f$ are true or false.

14. $f$ is concave up on the interval $(0, \infty)$ only.
   
a. True  
   b. False

15. $f$ is increasing on intervals $(-2, 0)$ and $(2, \infty)$.
   
a. True  
   b. False

16. $f(x)$ has a local maximum at $x = 0$.
   
a. True  
   b. False

17. $f$ has inflection points at $x = -1$ and $x = 1$.
   
a. True  
   b. False

6B
1. Let \( f(x) = x^{\sin x} \).

   a. Use logarithmic differentiation to find \( f'(x) \).

   \[
   f'(x) = \frac{\cos x}{x^{\sin x}}
   \]

   b. Find the slope of the tangent line to \( f(x) \) at \( x = \frac{3\pi}{2} \).

   \[
   m = \frac{\cos \left( \frac{3\pi}{2} \right)}{\left( \frac{3\pi}{2} \right)^{\sin \left( \frac{3\pi}{2} \right)}}
   \]
2. A boy standing at the end of a pier which is 12 feet above the water is pulling on a rope that is attached to a rowboat in the water. If he is pulling the rope in at 8 feet per minute, how fast is the boat docking (moving towards the pier) when it is 16 feet from the pier?

The boat is docking at __________ feet per minute.

3. To which of the functions below can we apply Rolle’s Theorem on the interval [0, 8]? Write “can apply” or “cannot apply”. If you can apply Rolle’s Theorem, find each value of c guaranteed by the theorem. If not, state a condition not satisfied by the function.

___________  a. \( f(x) = x^{2/3} - 2x^{1/3} \)

___________  b. \( f(x) = \frac{8x - x^2}{x - 1} \)
4. Let $f(x) = \sqrt{2x - 1}$.

   A. Find the linearization $L(x)$ of $f$ at $a = 5$.

   $L(x) =$

   B. Use $L(x)$ to approximate $\sqrt{9.18}$.  Hint: $\sqrt{9.18} = f(5.09)$.

   Write your answer as a decimal.
5. Let \( f(x) = 5x^6 - 6x^5 \).

A. Determine the following for \( f \) (if none, write “none”):

(a) \( x \)-intercept(s) ____________

(b) Show the signs of \( f' \) and \( f'' \) on the number lines below:

\[
\begin{align*}
\quad & f' \quad & f'' \\
\quad & \quad & \quad
\end{align*}
\]

(c) local maximum at \( x = \)_______ local maximum value: _______

(d) local minimum at \( x = \)_______ local minimum value: _______

(e) inflection point(s) at \( x = \) _______

B. Sketch the graph of \( y = 5x^6 - 6x^5 \), using the information above. Label all important features of the graph. Note that \( f \left( \frac{4}{5} \right) \approx -0.65 \).