MAC 2311  Spring 2010
Exam 4A

A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID number
   3) Discussion Section number

C. Under “special codes”, code in the test number 4, 1.
   1  2  3  •  5  6  7  8  9  0
   •  2  3  4  5  6  7  8  9  0

D. At the top right of your answer sheet, for “Test Form Code” encode A.
   •  B  C  D  E

E. This test consists of 9 five-point and 4 two-point multiple choice questions, bonus questions worth 6 points and two sheets (4 pages) of partial credit questions worth 27 points. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or proctor. Be prepared to show your picture ID with a legible signature.

3) The answers will be posted on the MAC 2311 homepage after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted on Elearning within one week.
Problems 1 - 9 are worth five points each.

1. Use l'Hospital's rule to evaluate \[ \lim_{x \to 0} \frac{e^{2x} - 2e^x + 1}{\cos(2x) - 1}. \]
   a. 2    b. 0    c. \(-\frac{1}{2}\)    d. -1    e. The limit does not exist.

2. A particle moves in a straight line so that its velocity at time \( t \) is \( t^2 - 2t \) feet per second. Find the total distance traveled by the particle on the time interval \( 0 \leq t \leq 4 \).
   a. \( \frac{32}{3} \) feet    b. 12 feet    c. 8 feet    d. \( \frac{16}{3} \) feet    e. 32 feet

3. Find the area of the largest rectangle which can be inscribed in a right triangle with legs of length 3 cm and 4 cm, if two sides of the rectangle lie along the legs. Hint: find the equation of the line.

   a. 3 cm\(^2\)    b. \( \frac{3}{2} \) cm\(^2\)    c. 4 cm\(^2\)    d. 5 cm\(^2\)    e. \( \frac{5}{2} \) cm\(^2\)
4. If \( F(x) = \int_1^\sqrt{x} \frac{t^2}{t^2 - 2} \, dt \), then \( F'(x) = \) ______.

a. \( \frac{\sqrt{x}}{2x - 4} \)  
b. \( \frac{x}{x - 2} \)  
c. \( \frac{x^{3/2}}{2x^2 - 4} \)  
d. \( \frac{\sqrt{x}}{x - 2} \)  
e. \( \frac{x - 1}{2x - 4} \)

5. Approximate the area of the region under \( f(x) = \frac{2}{x} \) on the interval \([1, 3]\)

with a Riemann sum using four subintervals of equal width and letting \( x_i^* \) be the left endpoint of the subinterval \([x_{i-1}, x_i]\).

a. \( \frac{1}{2} + \frac{5}{6} + \frac{2}{5} \)  
b. \( \frac{3}{2} + \frac{2}{3} + \frac{2}{5} \)  
c. \( 1 + \frac{5}{3} + \frac{4}{5} \)

d. \( 2 \ln 3 \)  
e. \( 3 + \frac{4}{3} + \frac{4}{5} \)

6. Find the absolute maximum and minimum values \( M \) and \( m \) of

\( f(x) = \sqrt[3]{x^2 - 1} \) on \([-1, 3]\) and use the Comparison Property
to find lower and upper bounds \( p \) and \( q \) for \( \int_{-1}^{3} f(x) \, dx \)

(that is, \( p \leq \int_{-1}^{3} \sqrt[3]{x^2 - 1} \, dx \leq q \)).

a. \( p = -1 \) and \( q = 4 \)

b. \( p = -4 \) and \( q = 0 \)

c. \( p = 0 \) and \( q = 8 \)

d. \( p = 0 \) and \( q = 4 \)

e. \( p = -4 \) and \( q = 8 \)
7. Suppose that \( f \) is a function so that \( f''(x) = -\sin x \), \( f'(\pi) = 2 \) and \( f(0) = -1 \). Find \( f\left(\frac{\pi}{2}\right) \).

a. 0   b. \( \frac{3\pi}{2} \)   c. -1   d. 3   e. \( \frac{3\pi}{2} - 2 \)

8. Find the area of the region bounded by \( f(x) = \begin{cases} 2x^2 + 1 & x < 0 \\ e^x & x \geq 0 \end{cases} \)
and the \( x \)-axis on the interval \([-1, \ln 2]\).

a. \( \frac{5}{3} \)   b. 1   c. \( \frac{8}{3} \)   d. \( \frac{10}{3} \)   e. 2

9. The population of a new settlement is growing at the rate
\[
\frac{dP}{dt} = 50t^{3/2} - 40t
\]
where \( P \) is the population of the settlement \( t \) years after it was founded. If 1000 people moved into the settlement initially, what is the population 4 years later?

a. 1340   b. 2880   c. 1960   d. 1320   e. 3720
Problems 10 – 16 are worth 2 points each.

10. Evaluate: \( \int \frac{(x - 3)^2}{x^2} \, dx = \) ________.

   a. \( 1 - \frac{6}{x} + \frac{9}{x^2} + C \)  
   b. \( -6 \ln |x| - \frac{9}{x} + C \)  
   c. \( x + \frac{6}{x^2} - \frac{9}{x^3} + C \)
   d. \( x - 6 \ln |x| - \frac{9}{x} + C \)  
   e. None of these

11. \( \int_0^{\pi/4} \frac{2 \cos^2 x - 1}{\cos^2 x} \, dx = \) ________.

   a. \( -1 \)  
   b. \( \frac{\pi}{2} - 1 \)  
   c. \( -\frac{\pi}{4} \)  
   d. \( \frac{\pi}{2} \)  
   e. 0

12. Evaluate \( \int_{-3}^0 |x + 1| \, dx \). You may want to sketch the region represented by the integral.

   a. \( \frac{5}{2} \)  
   b. 2  
   c. \( \frac{3}{2} \)  
   d. 3  
   e. 1

13. If \( \int_1^6 f(x) \, dx = 10 \) and \( \int_3^6 f(x) \, dx = 6 \), then \( \int_3^1 3f(x) \, dx = \) ________.

   a. 16  
   b. 4  
   c. \(-12\)  
   d. \(-16\)  
   e. 12
Bonus!! (Two points each)

14. Evaluate \( \lim_{{x \to \pi/2}} \frac{\cos x + x}{\sin x + 1} \).

   a. \( \frac{\pi}{2} \)  
   b. \( \infty \)  
   c. 0  
   d. \( -\infty \)  
   e. \( \frac{\pi}{4} \)

Use the graph of the function \( f \) sketched below to answer questions 15 and 16.

15. Which of the following is a possible graph of an antiderivative \( F \) of \( f \)?

   a.  
   b.  
   c.  

16. Suppose that \( \int_0^2 f(x) \, dx = \int_4^6 f(x) \, dx = 4 \) and \( \int_2^4 f(x) \, dx = -1 \).

   If \( F(0) = 2 \) for some antiderivative \( F \) of \( f \), then use the Fundamental Theorem to find \( F(6) \).

   a. 11  
   b. 5  
   c. 9  
   d. 7  
   e. 10
1. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $\frac{1}{2}$ inches, and the margins on the left and right are to be one inch. What should the dimensions of the page be so that the least amount of paper is used? Let $x$ and $y$ be the width and height of the print area as shown.

Function to be maximized _______________

Constraint _______________

\[
x = \underline{\ \ \ \ \ } \text{in.}
\]

\[
y = \underline{\ \ \ \ \ } \text{in.}
\]

Dimensions of the page: ___________ in. by ___________ in.

Use the Second Derivative Test to confirm your results.
2. If \( f(x) = \frac{x - 1}{e^x} \), then \( f'(x) = \frac{2 - x}{e^x} \) and \( f''(x) = \frac{x - 3}{e^x} \).

I. Determine the following. Leave function values in terms of \( e \). **If none, write “none”**.

(a) domain of \( f \) _________ vertical asymptote: \( x = \) _________

(b) Evaluate the limits, using L’Hospital’s Rule if necessary:

1) \( \lim_{x \to \infty} f(x) = \) ________________

2) \( \lim_{x \to -\infty} f(x) = \) ________________

(c) horizontal asymptote(s): \( y = \) ________________

(d) number lines indicating intervals of positive and negative values for \( f' \) and \( f'' \):

\[ \begin{align*}
\text{---------} & \quad f' \quad \text{---------} & \quad f'' \\
\end{align*} \]

(e) relative(local) maximum at \( x = \) ________________

maximum value: ________________

relative(local) minimum at \( x = \) ________________

minimum value: ________________

(f) inflection point(s): ________________

(g) intercept(s): ________________
2. (continued) II. Sketch the graph of \( y = \frac{x - 1}{e^x} \), using the information from part I. Clearly indicate all important features of the graph. Note: \( \frac{1}{e^2} \approx 0.14 \) and \( \frac{2}{e^3} \approx 0.10 \).

![Graph of the function y = (x-1)/e^x]

3. Use L’Hospital’s Rule to evaluate \( \lim_{x \to 0} (1 + 3 \tan x)^{2/x} \).

4. Find a Riemann Sum which approximates \( \int_0^3 (x^2 + 2) \, dx \) using 3 subintervals of equal width and letting \( x_i^* = \text{midpoint of the subinterval } [x_{i-1}, x_i] \).
5. Consider the area under the graph of \( f(x) = x^2 + 2 \) on \([0, 3]\).

(a) Set up a Riemann sum which approximates the area using \( n \) subintervals of equal width with \( x_i^* = \) right endpoint of the subinterval \([x_{i-1}, x_i]\).

\[
\sum_{i=1}^{n} \_\_\_\_\_\_
\]

(b) Find the exact area under the graph of \( f(x) = x^2 + 2 \) on \([0, 3]\) by evaluating the limit of the Riemann sum as \( n \to \infty \).

Note: \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2 \)

\[
\text{Area} = \_\_\_\_\_\_
\]
A. Sign your scantron sheet in the white area on the back in ink.

B. Write and code in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID number
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C. Under “special codes”, code in the test number 4, 2.
   1 2 3 ● 5 6 7 8 9 0
   1 ● 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code” encode B.
   A ● C D E

E. This test consists of 9 five-point and 4 two-point multiple choice questions, bonus questions worth 6 points and two sheets (4 pages) of partial credit questions worth 27 points. The time allowed is 90 minutes.

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1) Before turning in your test check for transcribing errors. Any mistakes you leave in are there to stay.

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3) The answers will be posted on the MAC 2311 homepage after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted on Elearning within one week.
Problems 1 - 9 are worth five points each.

1. Use L'Hospital's rule to evaluate \( \lim_{{x \to 0}} \frac{\cos(3x) - 1}{e^{3x} - 3e^x + 2} \).

   a. The limit does not exist.  
   b. \( \frac{3}{2} \)  
   c. 3  
   d. \(-\frac{1}{2}\)  
   e. 0

2. Find the area of the largest rectangle which can be inscribed in a right triangle with legs of length 4 cm and 3 cm, if two sides of the rectangle lie along the legs. Hint: find the equation of the line.

   a. 4 cm\(^2\)  
   b. \( \frac{3}{2} \) cm\(^2\)  
   c. 5 cm\(^2\)  
   d. \( \frac{5}{2} \) cm\(^2\)  
   e. 3 cm\(^2\)

3. If \( F(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^2 + 3} \, dt \), then \( F'(x) = \). 

   a. \( \frac{x + 1}{2x + 6} \)  
   b. \( \frac{\sqrt{x}}{x + 3} \)  
   c. \( \frac{\sqrt{x}}{2x + 6} \)  
   d. \( \frac{x^{3/2}}{2x^2 + 6} \)  
   e. \( \frac{x}{x + 3} \)
4. Find the absolute maximum and minimum values $M$ and $m$ of $f(x) = \sqrt[3]{x^2 - 1}$ on $[-1, 3]$ and use the Comparison Property to find lower and upper bounds $p$ and $q$ for $\int_{-1}^{3} f(x) \, dx$ (that is, $p \leq \int_{-1}^{3} \sqrt[3]{x^2 - 1} \, dx \leq q$).

   a. $p = -4$ and $q = 0$
   b. $p = 0$ and $q = 4$
   c. $p = 0$ and $q = 8$
   d. $p = -4$ and $q = 8$
   e. $p = -1$ and $q = 4$

5. A particle moves in a straight line so that its velocity at time $t$ is $t^2 - 2t$ feet per second. Find the total distance traveled by the particle on the time interval $0 \leq t \leq 6$.

   a. $\frac{116}{3}$ feet  
   b. 24 feet  
   c. 36 feet  
   d. $\frac{92}{3}$ feet  
   e. $\frac{58}{3}$ feet

6. Suppose that $f$ is a function so that $f''(x) = -\sin x$, $f'(\pi) = 2$ and $f(0) = -1$. Find $f\left(\frac{\pi}{2}\right)$.

   a. $\frac{3\pi}{2} - 2$  
   b. 3  
   c. $\frac{3\pi}{2}$  
   d. $-1$  
   e. 0
7. Approximate the area of the region under \( f(x) = \frac{2}{x} \) on the interval \([1, 3]\) with a Riemann sum using four subintervals of equal width and letting \( x_i^* \) be the left endpoint of the subinterval \([x_{i-1}, x_i]\).

\[
\begin{align*}
a. \quad & 3 + \frac{4}{3} + \frac{4}{5} \\
b. \quad & 1 + \frac{5}{3} + \frac{4}{5} \\
c. \quad & \frac{1}{2} + \frac{5}{6} + \frac{2}{5} \\
d. \quad & \frac{3}{2} + \frac{2}{3} + \frac{2}{5} \\
e. \quad & 2 \ln 3
\end{align*}
\]

8. The population of a new settlement is growing at the rate

\[
\frac{dP}{dt} = 50t^{3/2} - 40t \quad \text{where} \quad P \text{ is the population of the settlement at years after it was founded. If 2000 people moved into the settlement initially, what is the population 4 years later?}
\]

\[
\begin{align*}
a. \quad & 2320 \\
b. \quad & 4720 \\
c. \quad & 3880 \\
d. \quad & 2960 \\
e. \quad & 2340
\end{align*}
\]

9. Find the area of the region bounded by \( f(x) = \begin{cases} 2x^2 + 1 & x < 0 \\ e^x & x \geq 0 \end{cases} \) and the \( x \)-axis on the interval \([-1, \ln 2]\).

\[
\begin{align*}
a. \quad & \frac{10}{3} \\
b. \quad & \frac{8}{3} \\
c. \quad & 2 \\
d. \quad & \frac{5}{3} \\
e. \quad & 1
\end{align*}
\]
Problems 10 – 16 are worth 2 points each.

10. Evaluate: \( \int \frac{(x - 3)^2}{x^2} \, dx = \) 

a. \( x + \frac{6}{x^2} - \frac{9}{x^3} + C \)  
b. \( 1 - \frac{6}{x} + \frac{9}{x^2} + C \)  
c. \( x - 6 \ln |x| - \frac{9}{x} + C \)  
d. \( -6 \ln |x| - \frac{9}{x} + C \)  
e. None of these

11. Evaluate \( \int_{-3}^{0} |x + 1| \, dx \). You may want to sketch the region represented by the integral.

a. 2  
b. 3  
c. \( \frac{3}{2} \)  
d. 1  
e. \( \frac{5}{2} \)

12. \( \int_{0}^{\pi/4} \frac{2 \cos^2 x - 1}{\cos^2 x} \, dx = \) 

a. \( \frac{\pi}{2} \)  
b. \( -\frac{\pi}{4} \)  
c. 0  
d. \( \frac{\pi}{2} - 1 \)  
e. -1

13. If \( \int_{1}^{8} f(x) \, dx = 12 \) and \( \int_{4}^{8} f(x) \, dx = 8 \), then \( \int_{4}^{1} 2f(x) \, dx = \) 

a. 4  
b. 16  
c. 8  
d. -16  
e. -8
Bonus!! (Two points each)

14. Evaluate \( \lim_{x \to \pi/2} \frac{\cos x + x}{\sin x + 1} \).

   a. \( \frac{\pi}{4} \)   \quad b. 0 \quad c. \( \frac{\pi}{2} \) \quad d. \( \infty \) \quad e. \( -\infty \)

Use the graph of the function \( f \) sketched below to answer questions 15 and 16.

15. Which of the following is a possible graph of an antiderivative \( F \) of \( f \)?

   a. \hspace{2cm} \hspace{2cm} \hspace{2cm} b. \hspace{2cm} \hspace{2cm} \hspace{2cm} c.

16. Suppose that \( \int_0^2 f(x) \, dx = \int_4^6 f(x) \, dx = 4 \) and \( \int_2^4 f(x) \, dx = -1 \).

   If \( F(0) = 3 \) for some antiderivative \( F \) of \( f \), then use the Fundamental Theorem to find \( F(6) \).

   a. 9 \quad b. 12 \quad c. 9 \quad d. 7 \quad e. 10
1. A rectangular page is to contain 48 square inches of print. The margins at the top and bottom of the page are to be 2 inches, and the margins on the left and right are to be $1 \frac{1}{2}$ inches. What should the dimensions of the page be so that the least amount of paper is used? Let $x$ and $y$ be the width and height of the print area as shown.

Function to be maximized

Constraint

$$x = \text{_______ in.}$$

$$y = \text{_______ in.}$$

Dimensions of the page: _________ in. by _________ in.

Use the Second Derivative Test to confirm your results.
2. If \( f(x) = \frac{1-x}{e^x} \), then \( f'(x) = \frac{x-2}{e^x} \) and \( f''(x) = \frac{3-x}{e^x} \).

I. Determine the following. Leave function values in terms of \( e \). If none, write “none”.

(a) domain of \( f \) _________  vertical asymptote: \( x = \)_________

(b) Evaluate the limits, using L’Hospital’s Rule if necessary:

1) \( \lim_{x \to -\infty} f(x) = \)___________

2) \( \lim_{x \to \infty} f(x) = \)___________

(c) horizontal asymptote(s): \( y = \)___________

(d) number lines indicating intervals of positive and negative values for \( f' \) and \( f'' \):

\[ \text{___________} f' \quad \text{___________} f'' \]

(e) relative(local) maximum at \( x = \)___________

maximum value: _____________

relative(local) minimum at \( x = \)___________

minimum value: _____________

(f) inflection point(s): _____________

(g) intercept(s): _____________
2. (continued) II. Sketch the graph of \( y = \frac{1-x}{e^x} \), using the information from part I. Clearly indicate all important features of the graph. Note: \( \frac{1}{e^2} \approx -0.14 \) and \( \frac{2}{e^3} \approx -0.10 \).

3. Use L'Hospital's Rule to evaluate \( \lim_{x \to 0} (1 + 4\tan x)^{3/x} \).

4. Find a Riemann Sum which approximates \( \int_0^3 (x^2 + 5) \, dx \) using 3 subintervals of equal width and letting \( x_i^* = \text{midpoint of the subinterval } [x_{i-1}, x_i] \).
5. Consider the area under the graph of \( f(x) = x^2 + 5 \) on \([0, 3]\).

(a) Set up a Riemann sum which approximates the area using \( n \) subintervals of equal width with \( x_i^* \) = right endpoint of the subinterval \([x_{i-1}, x_i]\).

\[
\sum_{i=1}^{n}
\]

(b) Find the exact area under the graph of \( f(x) = x^2 + 5 \) on \([0, 3]\) by evaluating the limit of the Riemann sum as \( n \to \infty \).

Note: \( \sum_{i=1}^{n} i = \frac{n(n + 1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}, \sum_{i=1}^{n} i^3 = \left[ \frac{n(n + 1)}{2} \right]^2 \)

Area = 

10B