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1. Determine if the following are functions of $x$ or $t$, and if so, what is the domain of each?
   (a) $x^2 + y^2 = 4$
   (b) $\frac{2t}{t^2 - 1}$
   (c) $4 - x^2$

2. You want to fence in a rectangular area divided into two sections. To do this, you have $1200. If the middle fence is more expensive, say $4 per foot, while the outer fence costs only$2 per foot, find the dimensions which maximize your enclosed area. What is this area?

3. Expand and simplify.
   (a) $\log_2 \frac{\sqrt{a - 2}}{32}, a > 2$
   (b) $\ln \frac{x^5 e^{-3}}{\sqrt[3]{x(1 + x)^9}}$

4. Solve.
   (a) $4^{x+2} = \frac{1}{64}$
   (b) $\ln(x + 4) - \ln(x) = \ln(x - 2)$
5. Three years ago, you deposited $1000 in an account which compounds interest continuously. Today, you have $1500 in the account.

(a) Use these facts to find your annual interest rate.
(b) Now that you have the annual interest rate, figure out how much money the account will have in it 5 years from today.

6. A certain culture of bacteria grows exponentially in the lab. The culture starts with exactly 200 bacteria, and the population triples every two hours.

(a) How many bacteria cells will there be 12 hours after the start?
(b) How long will it take for bacteria population to hit 10 million?

7. For the function described by $f(x) = \sqrt{x^2}$:

(a) Find the average rate of change of the function over the interval [6,11].
(b) Find the average rate of change of the function over the interval $[x, x + h]$.
(c) Use limits to calculate the instantaneous rate of change.

8. Evaluate the following limits:

(a) $\lim_{x \to \infty} \frac{-4x^5 - 3x^2 + 1}{2x^4 + x^3 + x^2 + x + 1}$
(b) $\lim_{x \to \infty} \frac{3x^2 - 5x + 6}{7x^4 + 3x - 2}$
(c) $\lim_{x \to \infty} \frac{4x^3 + 9x^2 + 21}{3x^2 + 6x^3 - 2x}$
(d) $\lim_{x \to \infty} \frac{-2}{2 - 4e^{-x}}$
9. Refer to the graph to determine whether each statement is true or false.

(a) \( \lim_{x \to -3^+} f(x) = 2 \)
(b) \( \lim_{x \to 0} f(x) = 2 \)
(c) \( \lim_{x \to 2} f(x) = 1 \)
(d) \( \lim_{x \to 4^-} f(x) = 3 \)
(e) \( \lim_{x \to 4^+} f(x) \) does not exist.
(f) \( \lim_{x \to 4} f(x) = 2 \)

10. Using the same graph, identify and label all the types of discontinuities.

11. Consider the function:
\[
f(x) = \begin{cases} 
  |x + 2| & -3 \leq x < 1 \\
  3 & x = 1 \\
  2 + \sqrt{x} & x > 1 
\end{cases}
\]

(a) Find the domain of the function
(b) Use the limit definition of continuity to show that \( f(x) \) is continuous at \( x = 1 \)
12. The position, in meters, of a particle at time $t$ is given by:

$$s(t) = -9t^2 + 15t - 12$$

where $t$ is measured in seconds and is nonnegative ($t \geq 0$)

(a) What is the displacement from 0 to 8 seconds?

(b) Find the particle’s average rate of change from 2 to 4 seconds.

(c) Find a function for the instantaneous velocity at time $t$.

(d) What is the particle’s instantaneous velocity at 2 seconds?

13. Find the value of $k$ such that the following piecewise function is continuous for all real numbers.

$$f(x) = \begin{cases} 
  x^3 + k & x < 3 \\
  kx - 5 & x \geq 3
\end{cases}$$

14. Determine the discontinuities and their type for the following function. For any removable discontinuities, redefine $f(x)$ so that it is continuous at that point.

$$f(x) = \frac{x + 3}{x^2 - 5x - 24}$$