1) Find the value of \(a\) such that the line tangent to \(y = 2x^2 - \sqrt{x + 2}\) when \(x = 2\) is perpendicular to the line \(ax + 3y = 4\).

2) A spherical balloon has a radius changing at a rate \(r(t) = 2t + 1\). If the ball’s volume is modeled by \(V(r) = \frac{4}{3}\pi r^3\), how fast is the volume of the ball changing at time \(t=4\)? (The radius is measured in cm and time in seconds)

3) Evaluate the following limits:
   a. \(\lim_{x \to 4} \frac{2x^3 - 128}{\sqrt{x} - 2}\)
   b. \(\lim_{x \to 4} \frac{3 - \sqrt{x + 5}}{x - 4}\)
   c. \(\lim_{x \to 0} \frac{x^3 - 7x}{x^3}\)
   d. \(\lim_{x \to 6} \frac{x^2 + 2x - 3}{x^2 + 5x - 6}\)
   e. \(\lim_{x \to 2^+} \frac{x^2 - 6x + 8}{|x-2|}\)
   f. \(\lim_{x \to 2^-} \frac{x^2 - 6x + 8}{|x-2|}\)

4) Given the following function:
   \(f(x) = (x^2 - 2x + 2)^5\)
   a. Find \(f'\).
   b. Write the equation of the tangent line at \(x=0\).
5) Differentiate the following functions.
   
   a. \( f(x) = x\sqrt{x^2 - 3} \)
   
   b. \( g(x) = \frac{x^3 + 2}{x^2 + 1} \)
   
   c. \( h(x) = (3x^2 - 1)\left(x^2 - \frac{1}{x}\right) \)
   
   d. \( r(x) = \frac{x^3 + 2x^2 + x - 1}{\sqrt{x}} \)
   
   e. \( q(x) = \frac{x}{(2x + 1)^2} \)

6) A company sells “student enhancement” products. The cost of producing \( x \) units of “enhancement” products is modeled by the function

   \( C(x) = 400 + 50x + 0.03x^2 \)

   For any given cost function the average cost function is defined as

   \( \bar{C}(x) = \frac{C(x)}{x}, x \neq 0 \)

   a. Find the average cost function in this case.
   
   b. Find the average cost at a 100 units.
   
   c. Find the marginal cost at a 100 units and the marginal average cost at a 100 units.

7) Mary's Fudge sells square bottomed tin containers of delicious holiday fudge. The sides and bottom of the tin cost $1 per square inch but the special lid costs $1.50 per square inch. Mary would like to sell her fudge in 80 cubic inch size containers. What are the dimensions of the tin which will minimize cost?

8) In a new book, the publisher wants the printed area of the page to be 24 square inches, with half-inch margins on the sides of the page, and one-inch margins on the top and bottom. However, the author wants the total page size to be as small as possible. What dimensions should the pages be?

9) The demand function for a particular item is \( p(x) = \frac{-1}{50}x + 20 \). It costs the company 10 dollars to make each item, and the operating cost of its factory is 1000 dollars each day.

   a) Find the profit function \( P(x) \).
   
   b) How much should the company sell its item for in order to maximize profit?
   
   c) What is the maximum profit in a day?
10) The graph below represents the rate of change of a population of werewolves, \( W'(t) \) (in hundreds of werewolves) as a function of time, where \( t \) is measured in years. Use this graph to answer the following questions:

![Graph Image]

a) For what years after the first year and before the 7th year does the werewolf population reach a maximum? A minimum?

b) Depict number line charts for \( W' \) and \( W'' \). Use them to determine the intervals where \( W \) is increasing, decreasing, concave up, and concave down.

c) Give a possible graph for the werewolf population as a function of time.

11) Find the absolute minimum and maximum values for each of the following functions.

a) \( f(x) = \ln(x^2 + x + 1) \) over the interval \([-1,1]\)

b) \( f(t) = t\sqrt{4 - t^2} \) over the interval \([-1,2]\)

12) How many inflection points does the function have over the entire real number line? Where are they?

\[ g(x) = 3x^5 - 10x^4 + 10x^3 \]

13) Gravel is being dumped from a conveyor belt at a rate of 30 ft\(^3\)/min. As it falls from the conveyor belt, it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

14) Differentiate the following functions:

a. \( f(x) = 3(4x^2+2) \)

b. \( g(x) = \log_5\sqrt{5x + 2} \)

c. \( r(t) = 5\left(2\sqrt{t-2}\right) \)

d. \( q(x) = \ln\left(\frac{x}{2x-1}\right) \)
15) Consider the following function,

\[ f(x) = \frac{x^2}{(x - 2)^2} \]

List the locations of...

a) The vertical and horizontal asymptotes
b) The critical numbers, and the intervals where the function is increasing, decreasing, concave up, and concave down.

c) Local Minimum, local Maximum, and Points of Inflection
d) Sketch a graph of \( f(x) \).

16) Consider the following function

\[ f(x) = \frac{8}{x^3} + \frac{5}{x^3} \]

a) Find the absolute extrema on \([0,8]\)
b) Use the second derivative test to classify the type of the critical points for the function across the whole real line.
c) List the locations of the following:
   a. Vertical and Horizontal Asymptotes
   b. Critical numbers, and the intervals where the function is increasing, decreasing, concave up, and concave down
   c. Inflection Points
d) Sketch a graph of \( f(x) \).

17) Assume that the revenue function \( R \) and the cost function \( C \) for a business are given as:

\[ R(x) = 1163x - 9x^2 \]
\[ C(x) = 45x + 22 \]

where \( x \) is the daily production and sales. Assume that business is currently producing/selling 12 units per day and that production/selling are currently increasing 1 unit per day. Find the rate of change in profit per day.