## Skills

<table>
<thead>
<tr>
<th>Skill Code</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC1</td>
<td>Add, subtract, multiply, and divide real numbers</td>
<td>166</td>
</tr>
<tr>
<td>IC2</td>
<td>Apply the order of operations agreement</td>
<td>170</td>
</tr>
<tr>
<td>IIC1</td>
<td>Use properties of operations</td>
<td>172</td>
</tr>
<tr>
<td>IC3</td>
<td>Use scientific notation</td>
<td>175</td>
</tr>
<tr>
<td>IIIC2</td>
<td>Determine if a number is a solution to an equation or inequality</td>
<td>178</td>
</tr>
<tr>
<td>IC4</td>
<td>Use properties to identify equivalent equations and inequalities</td>
<td>180</td>
</tr>
<tr>
<td>IC5</td>
<td>Solve linear equations and inequalities</td>
<td>183</td>
</tr>
<tr>
<td>IC6</td>
<td>Use algebraic formulas</td>
<td>187</td>
</tr>
<tr>
<td>IC7</td>
<td>Find values of functions</td>
<td>189</td>
</tr>
<tr>
<td>IC8</td>
<td>Find factors of quadratic expressions</td>
<td>190</td>
</tr>
<tr>
<td>IC9</td>
<td>Identify specified regions of the coordinate plane</td>
<td>196</td>
</tr>
<tr>
<td>IIIC4</td>
<td>Find solutions to quadratic equations</td>
<td>193</td>
</tr>
<tr>
<td>IVC2</td>
<td>Solve a system of two linear equations in two unknowns</td>
<td>200</td>
</tr>
<tr>
<td>IIIC3</td>
<td>Solve problems involving the structure and logic of algebra</td>
<td>206</td>
</tr>
<tr>
<td>IVC1</td>
<td>Identify statements of proportionality and variation</td>
<td>209</td>
</tr>
<tr>
<td></td>
<td>Solve algebraic word problems with variables</td>
<td>212</td>
</tr>
</tbody>
</table>

## Practice Problems

217

## Practice Explanations

235
Algebra is a branch of mathematics that deals with expressions and equations that are combined according to the rules of arithmetic. Algebra skills help to organize your thoughts so that you can solve mathematical problems. A working knowledge of algebra is useful in many fields, such as physics, biology, astronomy, and social sciences.

The titles of the units found in this chapter are listed below in italics. Under each unit title is listed the specific algebra skill or skills covered in that unit.

*Add, subtract, multiply, and divide real numbers.*
Skill IC1a: The student will add and subtract real numbers.
Skill IC1b: The student will multiply and divide real numbers.

*Apply the order of operations agreement.*
Skill IC2: The student will apply the order-of-operations agreement to computations involving numbers and variables.

*Use properties of operations.*
Skill IIC1: The student will use properties of operations correctly.

*Use scientific notation.*
Skill IC3: The student will use scientific notation in calculations involving very large or very small measurements.

*Determine if a number is a solution to an equation or inequality.*
Skill IIC2: The student will determine whether a particular number is among the solutions of a given equation or inequality.

*Use properties to identify equivalent equations and inequalities.*
Skill IIC2: The student uses applicable properties to select equivalent equations and inequalities.

*Solve linear equations and inequalities.*
Skill IC4a: The student will solve linear equations.
Skill IC4b: The student will solve linear inequalities.

*Use algebraic formulas.*
Skill IC5: The student will use given formulas to compute results when geometric measurements are not involved.

*Find values of functions.*
Skill IC6: The student will find particular values of a function.
**Algebra Skills**

*Find factors of quadratic expressions.*
Skill IC7: The student will factor a quadratic expression.

*Find solutions to quadratic equations.*
Skill IC8: The student will find the roots of a quadratic equation.

*Solve a system of two linear equations in two unknowns.*
Skill IC9: The student will solve a system of two linear equations in two unknowns.

*Identify specified regions of the coordinate plane.*
Skill IIC4: The student will identify regions of the coordinate plane which correspond to specified conditions and vice versa.

*Solve problems involving the structure and logic of algebra.*
Skill IVC2: The student solves problems that involve the structure and logic of algebra.

*Identify statements of proportionality and variation.*
Skill IIC3: The student will recognize statements and conditions of proportionality and variation.

*Solve algebraic word problems with variables.*
Skill IVC1: The student solves real-world problems involving the use of variables, aside from commonly used geometric formulas.
ADD, SUBTRACT, MULTIPLY, AND DIVIDE REAL NUMBERS

Real numbers include rational numbers (terminating or repeating decimals) and irrational numbers (nonterminating, nonrepeating decimals). The operations of addition, subtraction, multiplication, and division of rational numbers are covered in the arithmetic section of this text. This section reviews these operations performed on irrational numbers such as $\pi$ and square roots of numbers that are not perfect squares. (At this point, it would be helpful to review the perfect squares such as 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ... and their square roots.) A square root is one of two equal factors of a number; for example, the square root of 16 is 4 ($\sqrt{16} = 4$) because $4^2$ or $4 \times 4 = 16$.

Addition and Subtraction of Irrational Numbers

As indicated in the paragraph above, irrational numbers are decimals that do not form a repeating pattern and do not terminate. The types of irrational numbers covered in this skill are $\pi$ (which is often approximated as 3.14) and square roots of numbers that are not perfect squares. The square root of a number can be written as a radical, in the form $\sqrt{a}$, consisting of a radical sign ($\sqrt{\text{}}$) and a radicand ($a$).

Addition and subtraction of irrational numbers are similar to addition and subtraction of variable terms in that you may only add or subtract like terms. Like radicals are radicals that have the same radicand. Add or subtract like radicals by combining their coefficients, leaving the radicand unchanged. For example, $4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$.

To add or subtract irrational expressions involving radicals, follow these steps:

Step 1. Simplify each radical separately to produce like radicals.

Step 2. Combine the coefficients of radicals with the same radicand.

Reminder: To simplify radicals, use the property: if $a$ and $b$ are positive, then $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. Factor the radicand so that one of the factors ($a$ or $b$) is a perfect square.

Examples

1. $2\sqrt{5} + \sqrt{20} =$

   Step 1. Simplify each radical separately to produce like radicals.

   $2\sqrt{5}$ cannot be simplified further, but $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$. 

---

166
Skill IC1  Add, subtract, multiply, and divide real numbers

Step 2. Combine the coefficients of radicals with the same radicand.

\[2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}\]

When adding or subtracting expressions involving \(\pi\), treat the \(\pi\) terms as like radicals. Add or subtract the coefficients and recopy \(\pi\).

2. \(3\pi + 6 + 2\pi = \)

The two terms involving \(\pi\) can be combined to obtain \(5\pi + 6\).

\textit{Caution:} \(3\pi + 2\pi\) does NOT equal \(5\pi^2\) and \(3\pi + 6 + 2\pi\) does NOT equal \(11\pi\).

Multiplication and Division of Irrational Numbers

When multiplying or dividing square roots, use the following steps:

\textbf{Step 1.} Place the radicands under the same radical sign. For example, \(\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}\) and

\[
\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}
\]

\textbf{Step 2.} Multiply or divide, as appropriate.

\textbf{Step 3.} Simplify the resulting radical so that the radicand does not have a perfect square as a factor.

\textbf{Examples}

3. \(\sqrt{3} \times \sqrt{8} = \)

\textit{Step 1.} Place the numbers under the same radical sign.

\[
\sqrt{3} \times \sqrt{8} = \sqrt{3 \times 8}
\]

\textit{Step 2.} Multiply.

\[
\sqrt{3 \times 8} = \sqrt{24}
\]

\textit{Step 3.} Simplify the resulting radical so that the radicand does not have a perfect square as a factor.

\[
\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}
\]
Skill IC1  

**Add, subtract, multiply, and divide real numbers**

Using another method, simplify each radical first. Then multiply or divide the coefficients and the radicals separately. Let's redo example 4 using this approach.

$$\sqrt{3} \times \sqrt{8} = \sqrt{3} \times (\sqrt{4} \times \sqrt{2}) = \sqrt{3} \times 2 \sqrt{2} = 2 \sqrt{3} \times 2 = 2 \sqrt{6}$$

In most cases, it is easier to multiply or divide the radicands first and then simplify the result as shown in the first solution of example 3.

4. \( \frac{\sqrt{36}}{\sqrt{3}} = \) \( \frac{\sqrt{36}}{\sqrt{3}} = \sqrt{\frac{36}{3}} \)

**Step 1.** Place the numbers under the same radical sign.

**Step 2.** Divide.

$$\sqrt{\frac{36}{3}} = \sqrt{12}$$

**Step 3.** Simplify the resulting radical so that the radicand does not have a perfect square as a factor.

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2 \sqrt{3}$$

**Division of Irrational Numbers**

Sometimes the division of the radicands does not result in an integer as it did in example 4. In such cases, follow these steps.

**Step 1.** Determine that the division of the radicands will not result in an integer.

**Step 2.** Rationalize the denominator by multiplying both the denominator and the numerator by a number that will make the denominator a perfect square.

**Step 3.** Simplify the numerator.
Skill IC1  Add, subtract, multiply, and divide real numbers

Examples

5. \( \frac{\sqrt{5}}{\sqrt{3}} = \)

**Step 1.** Determine that the division of the radicands will not result in an integer.

\[ \frac{5}{3} \]  
If 5 is divided by 3, the result is not an integer.

**Step 2.** Rationalize the denominator by multiplying both the denominator and the numerator by a number that will make the denominator a perfect square.

\[ \frac{\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{\sqrt{9}} = \frac{\sqrt{15}}{3} \]

**Step 3.** \( \frac{\sqrt{15}}{3} \) will not simplify further.

6. \( \frac{\sqrt{8}}{\sqrt{5}} = \)

**Step 1.** Determine that the division of the radicands will not result in an integer.

\[ \frac{8}{5} \]  
If 8 is divided by 5, the result is not an integer.

**Step 2.** Rationalize the denominator by multiplying both the denominator and the numerator by a number that will make the denominator a perfect square.

\[ \frac{\sqrt{8}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{40}}{\sqrt{25}} = \frac{\sqrt{40}}{5} \]

**Step 3.** Simplify the numerator.

\[ \frac{\sqrt{40}}{5} = \frac{\sqrt{4} \times \sqrt{10}}{5} = \frac{2\sqrt{10}}{5} \]
APPLY THE ORDER OF OPERATIONS AGREEMENT

When an expression contains a variety of arithmetic operations (exponents, multiplication, division, addition, subtraction), the order in which these operations are to be performed can be confusing.

For example: $12 + 6 ÷ 2$ could be computed as $18 ÷ 2 = 9$ or as $12 + 3 = 15$. Both results seem reasonable, but only one can be correct. So, which is it?

**Order of Operations**

To avoid confusion, a strict order of operations has been established. As a result, there is only one solution to each arithmetic or algebraic expression involving any of the operations listed above. The order of operations is as follows:

**Step 1.** Perform any operations inside the Parentheses.

**Step 2.** Evaluate expressions with Exponents.

**Step 3.** Multiply and/or Divide from left to right.

**Step 4.** Add and/or Subtract from left to right.

**Memory Device:** The capital letters in italics above form the pattern PEMDAS, which can easily be recalled by the phrase “Please Excuse My Dear Aunt Sally.”

**Caution:** As steps 3 and 4 above stress, each of these operations is performed from left to right as it occurs in the expression.

**Example**

1. $2 \times (2 + 6) - 8 ÷ 4 =$

   **Step 1.** Perform any operations inside the Parentheses.
   
   $2 \times (2 + 6) - 8 ÷ 4 = 2 \times 8 - 8 ÷ 4$.

   **Step 2.** Since this problem does not include Exponents, proceed to Step 3.

   **Step 3.** Multiply and/or Divide from left to right.
   
   $2 \times 8 - 8 ÷ 4 = 16 - 2$

   **Step 4.** Add and/or Subtract from left to right.
   
   $16 - 2 = 14$
Skill IC2  

Apply the order of operations agreement

When working with algebraic expressions involving a combination of numbers and variables, it is often necessary to distribute the multiplication over the addition/subtraction inside parentheses. (The "Distributive Property of Multiplication over Addition" is discussed in the unit on Skill IIC1 on pages 172–174.)

Examples

2. \[ 2(x + y) - \frac{4(2x - y)}{2} = \]

   \textit{Step 1.} Because no operations can be performed inside the Parentheses, proceed to Step 2.

   \textit{Step 2.} Because there are no expressions with Exponents, proceed to Step 3.

   \textit{Step 3.} Multiply and/or Divide from left to right.

   Distribute the 2 in the first term and simplify the fraction 4 over 2 in the same term.

   \[ 2(x + y) - \frac{4(2x - y)}{2} = 2x + 2y - 2(2x - y) \]

   Distribute \(-2\).

   \[ 2x + 2y - 2(2x - y) = 2x + 2y - 4x + 2y \]

   \textit{Step 4.} Add and/or Subtract from left to right.

   Combine like terms by adding or subtracting coefficients.

   \[ 2x + 2y - 4x + 2y = -2x + 4y \]

3. \[ (10 - 7)^2 + 5^2 = \]

   \textit{Step 1.} Perform any operations inside the Parentheses.

   \[ (10 - 7)^2 + 5^2 = 3^2 + 5^2 \]

   \textit{Step 2.} Evaluate expressions with Exponents.

   \[ 3^2 + 5^2 = 9 + 25 \]

   \textit{Step 3.} Multiply and/or Divide from left to right. This step is unnecessary.

   \textit{Step 4.} Add from left to right.

   \[ 9 + 25 = 34 \]
USE PROPERTIES OF OPERATIONS

Often in arithmetic it is helpful to change the order of terms before adding, or to group the order of factors before multiplying. For example, adding $2 + 7 + 8 + 3$ might be easier if you change the order and group the numbers that add up to 10. Look at this problem as $(2 + 8) + (7 + 3)$, which now clearly adds up to 20. This regrouping is possible because of certain properties of operations that will be discussed in this section. These properties are also needed in the study of algebra.

Properties of Operations

The properties of operations are rules that apply to addition and multiplication of real numbers. These properties allow the order of terms and factors to be changed, or regrouped, so that the original and resulting expressions will be equivalent. Some of these properties are included in the following table:

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>$a + b = b + a$</td>
<td>$ab = ba$</td>
</tr>
<tr>
<td>Associative</td>
<td>$a + (b + c) = (a + b) + c$</td>
<td>$a(bc) = (ab)c$</td>
</tr>
<tr>
<td>Identity</td>
<td>$a + 0 = a$</td>
<td>$(a)(1) = a$</td>
</tr>
<tr>
<td>Inverse</td>
<td>$a + (−a) = 0$</td>
<td>$a \times \frac{1}{a} = 1$  $(a \neq 0)$</td>
</tr>
</tbody>
</table>

Distributive property of multiplication over addition: $a(b + c) = ab + ac$

Using the table above, note the following:

- The **commutative property** allows you to change the order of terms or factors.
- The **associative property** allows you to regroup the terms or factors.
- The **identity properties** identify 0 as the additive identity and 1 as the multiplicative identity. They allow the number $a$ to retain its identity when $a$ and 0 are added or when $a$ and 1 are multiplied.
- The **inverse properties** identify $−a$ as the additive inverse of $a$ and $\left(\frac{1}{a}\right)$ as the multiplicative inverse of $a$, also known as the reciprocal of $a$.
- The **distributive property of multiplication over addition** establishes the technique for clearing grouping symbols (parentheses) without first performing the operation inside. For example, $2(x + 3) = 2x + 6$. 

172
Skill II C1

Use properties of operations

Examples

1. Identify the properties of operations illustrated by the following examples.
   
   A. \( 2 + (3 + 4) = (2 + 3) + 4 \)
   
   B. \( 2 + (3 + 4) = 2 + (4 + 3) \)
   
   C. \( 2(x + 4) = 2x + 8 \)
   
   D. \( 2(x + 4) = 2(4 + x) \)

   A. \( 2 + (3 + 4) = (2 + 3) + 4 \). On the left side of the equation, the 3 and 4 are grouped (associated). On the right side, the 2 and 3 are grouped. Therefore, this is an illustration of the \textit{associative property of addition}. 

   \textbf{Caution:} An equation involving three terms does not necessarily illustrate the associative property.

   B. \( 2 + (3 + 4) = 2 + (4 + 3) \). The numbers grouped inside the parentheses have not changed, but the order of the addition inside the parentheses has changed. Therefore, this is an illustration of the \textit{commutative property of addition}.

   C. \( 2(x + 4) = 2x + 8 \). Since the expression on the left involves multiplication over addition, it is likely to be the distributive property. Take care to examine the right side of the equation to make sure that the parentheses have been cleared; that is, the multiplication has been performed or at least expressed. Since the product of 2 and \( x \) is \( 2x \) and the product of 2 and 4 is 8, the multiplication was performed. Therefore, this is an illustration of the \textit{distributive property}.

   D. \( 2(x + 4) = 2(4 + x) \). The left side looks like the distributive property, but the parentheses were not cleared on the right side. Instead, the order of the addition inside the parentheses changed. Therefore, this is an illustration of the \textit{commutative property of addition}.

2. Identify the property illustrated by \( m^3 \left( \frac{1}{m^3} \right) = 1 \).

   On the left side \( m^3 \) is multiplied by its reciprocal, \( \frac{1}{m^3} \), producing 1 on the right side. Therefore, this is an illustration of the \textit{inverse property of multiplication}.

   \textbf{Reminder:} \( m^3 \cdot 1 = m^3 \) illustrates the \textit{identity property of multiplication}, since the "identity" of \( m^3 \) has been retained.
Skill IIIC1  

.Use properties of operations

Another type of problem dealing with these properties will ask you to identify an expression that is equivalent to the given expression. To determine the equivalent expression, view the given expression as one side of a property of operations and identify an expression that could serve as the other side.

3. Choose the expression equivalent to the following: \(3a + 3b\).
   A. \(6ab\)
   B. \(3a + b\)
   C. \(3(a + b)\)
   D. \(3 + (a + b)\)

   Option C is the only one that illustrates a property of operations correctly, namely the distributive property.

4. Choose the expression equivalent to the following: \(2a^2(a^2b^3)\).
   A. \(2(a^2a)b^3\)
   B. \(2a^2a^2 + 2a^2b^3\)
   C. \(3a^2b^3\)
   D. \((a^2b^3) + 2a^2\)

   Option A is the only one that illustrates a property of operations correctly, namely the associative property of multiplication.

5. Choose the statement that is NOT true for all real numbers.
   A. \(3x + 3y = 3(x + y)\)
   B. \((x + y)(x - y) = (x - y)(x + y)\)
   C. \(a + (-a) = 0\)
   D. \(3xy(2x + y) = 3xy(2y + x)\)

   **Reminder:** Except for one, all of the options given here are true for real numbers.

   Option D is the answer because it incorrectly attempts to illustrate the commutative property of addition inside the parentheses. \((2x + y)\) is not the same as \((2y + x)\). Option A correctly illustrates the distributive property. Option B correctly illustrates the commutative property of multiplication. Option C correctly illustrates the inverse property of addition.
**Use scientific notation**

**USE SCIENTIFIC NOTATION**

*Scientific notation* provides another way to write large or small decimal numbers. These decimal numbers, when expressed in scientific notation, can be multiplied or divided in a relatively easy manner using some basic rules of exponents. This section will review some of these rules of exponents and present a technique for conversion between decimal form and scientific notation.

**Scientific Notation** $b \times 10^n$

A number expressed in scientific notation will consist of two factors. The first factor, $b$, must be a decimal number greater than or equal to 1 and less than 10. The second factor will be a power of 10.

If the given number is less than 1, the power of 10, $n$, will be negative. If the given number is greater than or equal to 1, and less than 10, $n$ will be the zero power. If the given number is greater than 10, $n$ will be positive. The numerical value of the exponent, $n$, represents the number of places the decimal point had to be moved to obtain $b$.

<table>
<thead>
<tr>
<th>Decimal Form</th>
<th>Scientific Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0034</td>
<td>$3.4 \times 10^{-3}$</td>
<td>0.0034 is less than 1, so $n$ is a negative. Since the decimal point must be moved three places to the right to obtain 3.4, $n = -3$.</td>
</tr>
<tr>
<td>5.46</td>
<td>$5.46 \times 10^0$</td>
<td>5.46 is greater than or equal to 1 and less than 10, so $n$ is the zero power. Since the decimal point in 5.46 does not need to be moved, $n = 0$.</td>
</tr>
<tr>
<td>21,500</td>
<td>$2.15 \times 10^4$</td>
<td>21,500 is greater than 10, so $n$ is positive. Since the decimal point in 21,500 must be moved four places to the left, $n = 4$.</td>
</tr>
</tbody>
</table>

To convert from scientific notation to a decimal form, the *exponent* will indicate the number of places the decimal point must be moved. If the exponent, $n$, is negative, move the decimal point to the left to create a number less than 1. If $n$ is positive, move the decimal point to the right to create a number greater than 10.
**Skill IC3**

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Decimal Form</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.01 \times 10^{-4}$</td>
<td>.000201</td>
<td>Because $n = -4$, the decimal must be moved four places to the left. Note that zeros have been inserted as placeholders.</td>
</tr>
<tr>
<td>$3.22 \times 10^3$</td>
<td>3220 or 3,220</td>
<td>Because $n = 3$, the decimal point must be moved three places to the right.</td>
</tr>
</tbody>
</table>

**Examples**

1. Write 23,400,000 in scientific notation.

   Because 23,400,000 is greater than 10, the exponent, $n$, will be positive. Rewrite the number by placing the decimal point between the 2 and the 3. Since the decimal point must be moved seven places, $n = 7$.

   $$23,400,000 = 2.34 \times 10^7$$

2. Write $3.01 \times 10^{-5}$ as a decimal number.

   Since $n = -5$, the decimal point in 3.01 must be moved five places to the left. This will require that zeros be inserted as placeholders.

   $$3.01 \times 10^{-5} = .0000301 \text{ or } 0.0000301$$

**Review of Some Rules of Exponents**

**Rule 1:** $a^m \times a^n = a^{m+n}$

To multiply expressions with the same base ($a$), copy the base and add the exponents.

**Rule 2:** $\frac{a^m}{a^n} = a^{m-n}, \ a \neq 0$

To divide expressions with the same nonzero base, copy the base and subtract the denominator’s exponent from the numerator’s exponent.

**Rule 3:** $a^0 = 1, \ a \neq 0$

A nonzero base raised to an exponent of zero has a value of one.

When multiplying or dividing numbers in scientific notation, the rules of exponents will apply to the power of 10. The procedure for multiplication is to multiply the decimal numbers and then obtain a new power of 10 by Rule 1. The procedure for division is to divide the decimal numbers and then obtain a new power of 10 by Rule 2. In both cases, the result should be in scientific notation.
Skill IC3

Use scientific notation

Examples

3. \((2.3 \times 10^3) \times (4.0 \times 10^4) =\)
   
   Regroup the factors. \((2.3 \times 4.0) \times (10^3 \times 10^4)\)
   
   \(2.3 \times 4.0 = 9.2\) and \(10^3 \times 10^4 = 10^7\)
   
   \((2.3 \times 10^3) \times (4.0 \times 10^4) = 9.2 \times 10^7\)

4. \(\frac{2.4 \times 10^3}{4.8 \times 10^5} =\)
   
   \(\frac{2.4}{4.8} = 0.5\) and \(\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}\)
   
   \(\frac{2.4 \times 10^3}{4.8 \times 10^5} = 0.5 \times 10^{-2} = 5.0 \times 10^{-3}\)

   **Caution:** \(0.5 \times 10^{-2}\) is not in scientific notation. Since \(0.5 \times 10^{-2} = 0.005\), the answer expressed in scientific notation is \(5.0 \times 10^{-3}\).

5. \(0.0002 \times 23,000 =\)
   
   \(0.0002 \times 23,000 = 4.6\), which in scientific notation is \(4.6 \times 10^0\).

   The lengthy multiplication in example 5 can be avoided by converting the factors to scientific notation before multiplying.
   
   \(0.0002 = 2.0 \times 10^{-4}\) and \(23,000 = 2.3 \times 10^4\)
   
   \((2.0 \times 10^{-4}) \times (2.3 \times 10^4) = (2.0 \times 2.3) \times (10^{-4} \times 10^4) = 4.6 \times 10^0\)

6. \(\frac{0.00004}{2000} =\)
   
   \(0.00004 = 4.0 \times 10^{-5}\) and \(2000 = 2.0 \times 10^3\)
   
   \(\frac{0.00004}{2000} = \frac{4.0 \times 10^{-5}}{2.0 \times 10^3} = 2.0 \times 10^{-5-3} = 2.0 \times 10^{-8}\)

177
Skill IIC2  Determine if a number is a solution to an equation or inequality

DETERMINE IF A NUMBER IS A SOLUTION TO AN EQUATION OR INEQUALITY

An equation is a statement that sets two expressions equal. If an equation involves a single variable (unknown), then a solution is a number that makes the equation true when substituted for the variable. An equation may have no solutions, exactly one solution, or more than one solution.

Examples

1. Determine whether \( x = 3 \) is a solution for the statement \( x^2 - 1 = 2x + 2 \).

   Substitute 3 for \( x \) and simplify each side separately.

   \[
   \begin{array}{c|c}
   x^2 - 1 & 2x + 2 \\ 
   3^2 - 1 & 2(3) + 2 \\ 
   9 - 1 & 6 + 2 \\ 
   8 & 8 \\
   \end{array}
   \]

   Since 8 = 8, \( x = 3 \) is a solution.

2. Determine whether \( x = -2 \) is a solution for the statement \( |3x + 5| = x + 1 \).

   Substitute -2 for \( x \) and simplify each side separately.

   \[
   \begin{array}{c|c}
   |3x + 5| & x + 1 \\ 
   |3(-2) + 5| & -2 + 1 \\ 
   |-6 + 5| & -1 \\ 
   |-1| & -1 \\ 
   1 & -1 \\
   \end{array}
   \]

   Since 1 \( \neq -1 \), \( x = -2 \) is not a solution.

An inequality states that two quantities are not necessarily equal. Recall the inequality symbols \( <, >, \leq, \text{ and } \geq \). If an inequality involves a single variable, then a solution is a number that makes the inequality true when substituted for the variable.
Skill IIC2 Determine if a number is a solution to an equation or inequality

3. Determine whether $x = 5$ is a solution for the statement $(x - 5)(x + 4) \geq 0$.

Substitute 5 for $x$ and simplify each side separately.

\[
\begin{array}{c|c}
(5 - 5)(5 + 4) & 0 \\
(0)(9) & 0 \\
0 & 0 \\
\end{array}
\]

Since $0 \geq 0$, $x = 5$ is a solution.

**Caution:** A statement such as $0 \geq 0$ is sometimes incorrectly perceived as false because the $>$ part of the statement is false. Recall that the symbol $\geq$ means *greater than OR equal to*. If either one of the conditions $>$ or $=$ is satisfied, the inequality is TRUE.

4. Determine whether $x = \frac{1}{2}$ is a solution for the statement $-x^2 + 6x > 4x + 1$.

Substitute $\frac{1}{2}$ for $x$ and simplify each separately.

\[
\begin{array}{c|c}
-\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) & 4\left(\frac{1}{2}\right) + 1 \\
-\frac{1}{4} + 3 & 2 + 1 \\
2\frac{3}{4} & 3 \\
\end{array}
\]

Since $2\frac{3}{4}$ is not greater than 3, $x = \frac{1}{2}$ is not a solution.
USE PROPERTIES TO IDENTIFY EQUIVALENT EQUATIONS AND INEQUALITIES

Two equations or inequalities are equivalent if they have the same solution(s). The equation \(2x + 1 = 11\) is equivalent to the equation \(2x = 10\) because they have the same solution, \(x = 5\). The properties used to obtain equivalent equations or inequalities are summarized below.

Properties of Equality and Inequality

For rational numbers \(a, b, c:\)

Property i. If \(a = b\), then \(a + c = b + c\).
Property ii. If \(a > b\), then \(a + c > b + c\).
Property iii. If \(a = b\), then \(ac = bc\). (For \(c = 0\), \(a = b\) is not necessarily equivalent to \(ac = bc\).)
Property iv. For \(c > 0\), if \(a > b\), then \(ac > bc\).
Property v. For \(c < 0\), if \(a > b\), then \(ac < bc\).
Property vi. If \(a > b\) and \(b > c\), then \(a > c\).

Properties i and ii establish that when the same quantity is added to both sides of an equation or inequality, the result will be an equivalent equation or inequality. Similarly, property iii establishes that when the same nonzero quantity is multiplied on both sides of an equation, the result will be an equivalent equation. Properties iv and v should be studied together. They establish that when the same positive quantity is multiplied on both sides of an inequality, the result is an equivalent inequality; but when the same negative quantity is multiplied on both sides of an inequality, the inequality symbol must be reversed to produce an equivalent inequality. Property vi establishes that where one quantity is greater than a second quantity and the second quantity is greater than a third, then the first quantity is also greater than the third. Conversely, if the first quantity is smaller than a second and the second quantity is smaller than a third, then the first quantity will also be smaller than the third quantity.

Examples

1. Find an equivalent equation for \(2x + 1 = 11\).

\[
2x + 1 = 11
\]

By property i, \(2x + 1 + (-1) = 11 + (-1)\).

\[
2x = 10
\]

Therefore, an equivalent equation is \(2x = 10\).
Skill IIIC2 Use properties to identify equivalent equations and inequalities

2. Find an equivalent inequality for $8x - 7 > 4 - 2x$.

\[ 8x - 7 > 4 - 2x \]

By property ii, $8x - 7 + 2x > 4 - 2x + 2x$.

\[ 10x - 7 > 4 \]

Therefore, an equivalent equation is $10x - 7 > 4$.

3. Find an equivalent equation for $2x - 1 = -\frac{1}{6}x$.

\[ 2x - 1 = -\frac{1}{6}x \]

By property iii, \((-6) (2x - 1) = (-6) \left( -\frac{1}{6}x \right)\).

\[ -12x + 6 = x \]

Therefore, an equivalent equation is $-12x + 6 = x$.

4. Given $y > 0$, find the inequality that is equivalent to $\frac{5x}{y} < y + 1$ when both sides are multiplied by $y$.

\[ \frac{5x}{y} < y + 1 \]

By property iv, \((y) \left(\frac{5x}{y}\right) < (y + 1)(y)\).

\[ 5x < y^2 + y \]

Therefore, an equivalent inequality is $5x < y^2 + y$.

Reminder: The inequality symbol remained the same because both sides were multiplied by the positive value $y$. 
Skill IIIC2 Use properties to identify equivalent equations and inequalities

5. Find an equivalent inequality for \(-3x < 21\).

\[-3x < 21\]

By property v, \((-\frac{1}{3})(-3x) > (-\frac{1}{3})(21)\).

\[x > -7\]

Therefore, an equivalent inequality is \(x > -7\).

Reminder: Since both sides are multiplied by a negative value, the inequality symbol is reversed.

An inequality of the form \(a < x < b\) is called a compound inequality. In this case, properties ii, iv, and v must be applied to all three parts of the inequality.

6. Find an equivalent inequality for \(-5 < x - 2 < 11\).

\[-5 < x - 2 < 11\]

By property ii, \(-5 + 2 < x - 2 + 2 < 11 + 2\).

\[-3 < x < 13\]

Therefore, an equivalent inequality is \(-3 < x < 13\).

7. Find an equivalent inequality for \(-1 < -4x < 8\).

\[-1 < -4x < 8\]

By property v, \((-\frac{1}{4})(-1) > (-\frac{1}{4})(-4x) > (-\frac{1}{4})(8)\).

\[\frac{1}{4} > x > -2 \text{ or } -2 < x < \frac{1}{4}\]

Therefore, an equivalent inequality is \(\frac{1}{4} > x > -2 \text{ or } -2 < x < \frac{1}{4}\).

8. Fill in the blank to complete a true statement: “If \(x > y\) and \(y > 7\), then _____.”

By property vi, “If \(x > y\) and \(y > 7\), then \(x > 7\).”
SOLVE LINEAR EQUATIONS AND INEQUALITIES

Recall that a solution is a number that makes an equation or inequality true when substituted for the variable. Remember also that the product of a number and its reciprocal or multiplicative inverse is 1. For example, \( \frac{4}{5} \times \frac{5}{4} = 1 \). The reciprocal of \( \frac{4}{5} \) is \( \frac{5}{4} \) and the reciprocal of \( \frac{5}{4} \) is \( \frac{4}{5} \).

To find the solution(s) for an equation, follow the steps below to isolate the variable on one side of the equation.

**Step 1.** Remove grouping symbols ( ) or [ ], using the distributive property (see Skill IIIC2).

**Step 2.** Combine like terms on the same side of the equal sign.

**Step 3.** Express the equation with the variable term on one side and the constant term on the other side.

**Step 4.** Multiply both sides of the equation by the multiplicative inverse or reciprocal of the numerical coefficient.

**Reminder:** You should check the apparent solution by substituting it in the original equation.

**Examples**

1. Solve for \( x \): \( 4(x - 1) + 2x = 14 - (3 - x) \).

   **Step 1.** Remove grouping symbols ( ) or [ ], using the distributive property.
   
   \[ 4x - 4 + 2x = 14 - 3 + x \]

   **Reminder:** The expression \( -(3 - x) \) means \( -1(3 - x) \). The \(-1\) should be distributed, or applied to both sides of the equation, to obtain \(-3 + x\).

   **Step 2.** Combine like terms on the same side of the equal sign.

   On the left side, the like terms are \( 4x + 2x \), which are combined to produce \( 6x \).
   On the right side, \( 14 \) and \( -3 \) are like terms, which are combined to produce \( 11 \).

   \[ 6x - 4 = 11 + x \]

   **Step 3.** Express the equation with the variable term on one side and the constant term on the other side.

   The objective is to have the variable term on one side and the constant term on the other side of the equation. To move a term from one side of the equation to the other, add the additive inverse of this term to both sides of the equation.
Skill IC4  Solve linear equations and inequalities

\[ 6x + (-x) - 4 = 11 + x + (-x) \]
\[ 5x - 4 = 11 \]
\[ 5x - 4 + 4 = 11 + 4 \]
\[ 5x = 15 \]

**Step 4.** Multiply both sides of the equation by the multiplicative inverse or reciprocal of the numerical coefficient.

\[ \left( \frac{1}{5} \right)(5x) = \left( \frac{1}{5} \right)(15) \]
\[ x = 3 \]

Check the apparent solution by substituting it for the variable wherever the variable appears in the original equation.

<table>
<thead>
<tr>
<th>[4(x - 1) + 2(x)]</th>
<th>[14 - (3 - x)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4(3 - 1) + 2(3)]</td>
<td>[14 - (3 - 3)]</td>
</tr>
<tr>
<td>[4(2) + 2(3)]</td>
<td>[14 - 0]</td>
</tr>
<tr>
<td>8 + 6</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>14 (true)</td>
</tr>
</tbody>
</table>

The solution is \(x = 3\).

**Reminder:** Steps 1–4 are applied only as needed. For instance, if an equation has no grouping symbols, the procedure starts with Step 2. If there are no like terms on the same side of the equal sign, proceed to the next step.

2. Solve for \(x\): \(-4 = 6 - x\).

**Steps 1.** There are no grouping symbols or like terms on the same side of the equal sign; therefore, steps 1 and 2 are not necessary.

**Step 3.** Express the equation with the variable term on one side and the constant term on the other side.

\[-4 - 6 = 6 - 6 - x\]
\[-10 = -x\]

**Step 4.** Multiply both sides of the equation by the multiplicative inverse or reciprocal of the numerical coefficient.

\[(-1)(-10) = (-1)(-x)\]
\[10 = x\]

The solution is \(x = 10\).
Skill IC4  Solve linear equations and inequalities

3. Solve for m: \(2(6m - 2) - (m + 4) = 5\).

\textit{Step 1.} Remove grouping symbols ( ) or [ ], using the distributive property.
\[12m - 4 - m - 4 = 5\]

\textit{Step 2.} Combine like terms on the same side of the equal sign.
\[11m - 8 = 5\]

\textit{Step 3.} Express the equation with the variable term on one side and the constant term on the other side.
\[11m - 8 + 8 = 5 + 8\]
\[11m = 13\]

\textit{Step 4.} Multiply both sides of the equation by the multiplicative inverse or reciprocal of the numerical coefficient.
\[\left(\frac{1}{11}\right)(11m) = \left(\frac{1}{11}\right)(13)\]
\[m = \frac{13}{11}\]

The solution is \(m = \frac{13}{11}\).

\textit{Reminder:} This procedure also works for inequalities, with one reminder. In Step 4, if both sides of an inequality are multiplied or divided by a negative quantity, the inequality symbol must be reversed.

4. Solve for a: \(5(1 - a) - 3 < 7\).

\textit{Step 1.} Remove grouping symbols ( ) or [ ], using the distributive property.
\[5 - 5a - 3 < 7\]

\textit{Step 2.} Combine like terms on the same side of the equal sign.
\[-5a + 2 < 7\]

\textit{Step 3.} Express the equation with the variable term on one side and the constant term on the other side.
\[-5a + 2 - 2 < 7 - 2\]
\[-5a < 5\]
Step 4. Multiply both sides of the equation by the multiplicative inverse or reciprocal of the numerical coefficient.

\[
\left( \frac{-1}{5} \right)(-5a) > \left( \frac{-1}{5} \right)(5)
\]

\[a > -1\]

The solution is \(a > -1\).

5. Solve for \(x\): \(4 - (x + 1) > 3(1 - x) + x\).

Step 1. Remove grouping symbols ( ) or [ ], using the distributive property.

\[4 - x - 1 > 3 - 3x + x\]

Step 2. Combine like terms on the same side of the equal sign.

\[-x + 3 > 3 - 2x\]

Step 3. Express the equation with the variable term on one side and the constant term on the other side.

\[-x + 2x + 3 > 3 - 2x + 2x\]

\[x + 3 > 3\]

\[x + 3 - 3 > 3 - 3\]

\[x > 0\]

Step 4. Because the numerical coefficient of \(x\) is already 1, multiplication is not necessary.

\[x > 0\]

The solution is \(x > 0\).
USE ALGEBRAIC FORMULAS

An algebraic formula establishes a relationship between two or more variables. For example, the formula to compute interest to be paid on a loan is \( I = PRT \), where \( I \) = interest, \( P \) = principal, \( R \) = rate of interest per year, and \( T \) = time of loan expressed in years. \( P, R, \) and \( T \) will vary from one loan to another. The value will depend on the values substituted for \( P, R, \) and \( T \). To solve problems involving algebraic formulas, follow the steps below.

**Step 1.** Identify the given values.

**Step 2.** Substitute the given values into the formula.

**Step 3.** Solve for the unknown variable.

**Examples**

1. Given the formula to calculate interest, \( I = PRT \), compute the amount of interest that would be paid on a loan of $1000 issued at a rate of 10% for a period of 2 years.

   **Step 1.** Identify the given values. The given values are \( P = $1000, R = 10\% \), which equals 0.10, and \( T = 2 \).

   **Step 2.** Substitute the given values into the formula.

   \[
   I = (1000)(0.10)(2)
   \]

   **Step 3.** Solve for the unknown variable.

   \[
   I = 200
   \]

   Interest that would be paid on the loan is $200.

2. Given the interest formula, \( I = PRT \), compute the time in years required to earn $100 interest on an investment of $2500 at a rate of 8%.

   **Step 1.** Identify the given values. The given values are \( I = $100, P = $2500, \) and \( R = 8\% \), which equals 0.08.

   **Step 2.** Substitute the given values into the formula.

   \[
   100 = (2500)(0.08)T
   \]
Skill IC5

Use algebraic formulas

Step 3. Solve for the unknown variable.

\[ 100 = 200T \]

\[ \frac{100}{200} = T \]

\[ T = \frac{1}{2} \]

The time required to earn $100 interest is \( \frac{1}{2} \) year.

3. The formula for converting Celsius temperature to Fahrenheit temperature is \( F = \frac{9}{5}C + 32 \). Convert the Celsius temperature 30° C to Fahrenheit.

Step 1. Identify the given values.

The given value is \( C = 30° \).

Step 2. Substitute the given values into the formula.

\[ F = \frac{9}{5}(30) + 32 \]

Step 3. Solve for the unknown variable.

\[ F = \frac{9}{5}(30) + 32 \]

\[ F = 54 + 32 \]

\[ F = 86° \]

The Celsius temperature 30° C is equal to the Fahrenheit temperature 86° F.
FIND VALUES OF FUNCTIONS

The idea of a function is an important concept in mathematics because of its many applications in other fields of study. A function establishes a relationship between one set known as the domain and a second set known as the range, such that each element of the domain corresponds to one and only one element of the range.

There is more than one way to express a function. One way is to write each domain element, x, with its corresponding range element, y, as a set of ordered pairs in the form (x, y). For example, the set \{(1, 2), (2, 4), (3, 6)\} is a function that relates the domain elements \{1, 2, 3\} to the range elements \{2, 4, 6\} such that each domain element corresponds to one and only one range element. The set of ordered pairs, \{(1, 2), (2, 4), (1, 3)\}, is not a function because the domain element 1 corresponds to two different range elements, 2 and 3.

Another way to express a function is by using an equation. This equation serves to describe the operations that must be performed on the domain element to obtain its corresponding range element. For example, \(y = 2x\) indicates that for each \(x\)-value substituted into the equation, the corresponding \(y\)-value is produced by multiplying the \(x\)-value by 2. This function consists of infinitely many ordered pairs such as \(\ldots (-1, -2), \left(\frac{1}{2}, 1\right), (2, 4)\).

For functions, it is conventional to use the notation \(f(x)\) to represent the range element. This notation is known as function notation and is read “\(f\) of \(x\).” In effect, \(y\) and \(f(x)\) are interchangeable.

Functional notation directs you to substitute a given value for the domain variable. For instance, \(f(2)\), read “\(f\) of 2,” directs you to substitute 2 for each \(x\) in the equation and calculate the range value.

**Examples**

1. Given \(f(x) = 2x + 1\), find \(f(2)\).

   \(f(2)\) directs you to substitute 2 for every \(x\) in the equation \(f(x) = 2x + 1\).

   \[f(2) = 2(2) + 1 = 5\]

2. Given \(f(x) = -x^2 + 2x - 3\), find \(f(-1)\).

   \(f(-1)\) directs you to substitute -1 for every \(x\) in the equation \(f(x) = -x^2 + 2x - 3\).

   \[
f(-1) = -(-1)^2 + 2(-1) - 3 = -1 + (-2) - 3 = -6
   
   **Caution:** Be careful with the order of operation with \(-x^2\). The \(x\)-value should be squared before it is negated.
FIND FACTORS OF QUADRATIC EXPRESSIONS

A factor is any number of two or more numbers that when multiplied together form a product. Factoring involves finding the factors of an expression over a specified factor set. A quadratic expression has the "standard form," \( ax^2 + bx + c \), where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \). (The reason \( a \) is not allowed to equal 0 is to assure that there will always be a squared term.) The objective of this section is to review how to factor a quadratic expression. You will be given a quadratic expression and asked to find a linear factor, which is an expression of the form \( dx + e \), where \( d \) and \( e \) are real numbers and \( d \neq 0 \).

One way to find a single linear factor is to express the quadratic as a product of its linear factors. For example, \( x^2 - x - 6 \) can be expressed \((x - 3)(x + 2)\). This factored form contains two linear factors, \( x - 3 \) and \( x + 2 \). The remainder of this section will serve as an explanation of the most popular technique for factoring quadratic expressions in the form \( ax^2 + bx + c \).

Factoring a Quadratic Using the "FOIL" Technique

The word FOIL serves as a device to help the student remember how to multiply two linear factors such as \((2x + 1)(3x - 2)\).

\[
\begin{align*}
\text{First, multiply the First terms of each factor.} & \quad (2x)(3x) = 6x^2 \\
\text{Then, multiply the Outside terms of each factor.} & \quad (2x)(-2) = -4x \\
\text{Next, multiply the Inside terms of each factor.} & \quad (1)(3x) = 3x \\
\text{Finally, multiply the Last terms of each factor.} & \quad (1)(-2) = -2
\end{align*}
\]

Reminder: The outside and inside terms will combine to obtain the middle term. \((-4x + 3x = -x)\)

The resulting quadratic expression is \(6x^2 - 4x + 3x - 2\), which simplifies to \(6x^2 - x - 2\).
Setting up the product \((3x - 3)(5x - 2)\) as follows will illustrate the ease of the FOIL method.

\[
(3x - 3)(5x - 2) \quad \frac{15x^2}{\frac{6}{\frac{15x}{\frac{-15x}{\frac{-6x}{}}}}} \\
\text{First terms} \quad (3x)(5x) = 15x^2 \\
\text{Outside terms} \quad (3x)(-2) = -6x \\
\text{Inside terms} \quad (-3)(5x) = -15x \\
\text{Last terms} \quad (-3)(-2) = 6
\]

\(15x^2 - 6x - 15x + 6 = 15x^2 - 21x + 6\)

The FOIL method converts the product of two linear factors into a quadratic expression. The key to factoring a quadratic is to reverse this method and determine the first and last terms of the linear factors that will produce the first and last terms of the quadratic while at the same time producing outside and inside terms that will combine to produce the middle term of the quadratic. To get the correct factors involves some trial and error, which gets easier with practice. The key lies in FOILing the factors to see if they produce the given quadratic.
Skill IC7

Find factors of quadratic expressions

Steps involved in factoring a quadratic expression in standard form are as follows:

**Step 1.** Determine all possible pairs of factors of the first term of the quadratic. Choose a particular pair to serve as the first terms in the linear factors.

**Step 2.** Determine all possible pairs of factors of the last term of the quadratic. Choose a particular pair to serve as the last terms in the linear factors.

**Step 3.** Compute the outside and inside products to see if their sum produces the middle term of the quadratic. If not, try rearranging these terms or start over with new pairs. This is where the trial and error of this procedure occurs.

**Reminder:** If the last term of the quadratic is positive, its factors are either both positive or both negative. The sign of the middle term will determine which sign to use.

Examples

1. Find the linear factors of the quadratic expression $2x^2 + 5x + 2$.

   **Step 1.** Determine all possible pairs of factors of the first term of the quadratic. Choose a particular pair to serve as the first terms in the linear factors.
   
The factors of $2x^2$ are $2x$ and $x$ or $-2x$ and $-x$.

   **Step 2.** Determine all possible pairs of factors of the last term of the quadratic. Choose a particular pair to serve as the last terms in the linear factors.
   
The pairs of factors whose product is 2 are 1 and 2, or $-1$ and $-2$.

   **Step 3.** Compute the outside and inside products to see if their sum produces the middle term of the quadratic. If not, try rearranging these terms or start over with new pairs. This is where the trial and error of this procedure occurs.
   
   Put these factors into linear expressions and use the FOIL technique to see which set of expressions produces the quadratic.
   
   $(2x + 1)(x + 2)$
   $2x^2 + 4x + 1x + 2$
   $2x^2 + 5x + 2$

   Therefore, one possible linear factor is $2x + 1$, and the other is $x + 2$. 
Skill IC7

**Find factors of quadratic expressions**

2. Find a linear factor of the quadratic expression \(3x^2 + 10x - 8\).

*Step 1.* Determine all possible pairs of factors of the first term of the quadratic. Choose a particular pair to serve as the first terms in the linear factors.

The factors of \(3x^2\) are \(3x\) and \(x\) or \(-3x\) and \(-x\).

*Step 2.* Determine all possible pairs of factors of the last term of the quadratic. Choose a particular pair to serve as the last terms in the linear factors.

The pairs of factors whose product is \(-8\) are \(2\) and \(-4\), \(-2\) and \(4\), \(1\) and \(-8\), or \(-1\) and \(8\).

*Step 3.* Put these factors into linear expressions and use the FOIL technique to see which set of expressions produces the quadratic.

\((3x + 8)(x - 1)\) has a middle term of \(-3x + 8x = 5x\), which is incorrect.

\((3x + 4)(x - 2)\) has a middle term of \(-6x + 4x = -2x\), which is incorrect.

\((3x - 2)(x + 4)\) has a middle term of \(12x - 2x = 10x\), which is correct.

Therefore, one possible linear factor is \(3x - 2\), and the other factor is \(x + 4\).

3. Which is a linear factor of the following expression?

\(4x^2 - 9x + 2\)

A. \(4x - 1\)

B. \(2x - 2\)

C. \(2x - 1\)

D. \(x + 2\)

*Step 1.* Determine all possible pairs of factors of the first term of the quadratic. Choose a particular pair to serve as the first terms in the linear factors.

The factors of \(4x^2\) are \(4x\) and \(x\), \(-4x\) and \(-x\), \(2x\) and \(2x\), or \(-2x\) and \(-2x\).

*Step 2.* Determine all possible pairs of factors of the last term of the quadratic. Choose a particular pair to serve as the last terms in the linear factors.

Factors of \(2\) are \(1\) and \(2\), or \(-1\) and \(-2\). Since the middle term \((-9x)\) is negative, use negative factors.

*Step 3.* Put these factors into linear expressions and use the FOIL technique to see which set of expressions produces the quadratic.

\((4x - 1)(x - 2)\) has a middle term of \(-8x - x = -9x\), which is correct.

Therefore, A is the correct answer.
FIND SOLUTIONS TO QUADRATIC EQUATIONS

The previous section reviewed the concept of factoring a quadratic expression, given in standard form \( ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers and \( a \neq 0 \). We now turn our attention to solving an equation in the form \( ax^2 + bx + c = 0 \), called a \textit{quadratic equation}. To solve quadratic equations, substitute the numerical values for \( a \), \( b \), and \( c \) into the \textit{quadratic formula},

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

The steps for solving quadratic equations are outlined below.

\textit{Step 1:} Express the equation in standard form \( ax^2 + bx + c = 0 \).

\textit{Step 2:} Determine whether the quadratic can be readily factored.

\textit{Step 3:} If the quadratic can be readily factored:
\begin{itemize}
  \item factor the quadratic
  \item set each factor equal to zero
  \item solve each equation to find the solution(s) or root(s)
\end{itemize}

If the quadratic cannot be readily factored:
\begin{itemize}
  \item substitute the numerical values for \( a \), \( b \), and \( c \) into the quadratic formula
    \[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
    \]
  \item simplify this expression
\end{itemize}

\textit{Reminder:} The quadratic formula may produce 0, 1, or 2 solutions or roots, depending on the value of \( b^2 - 4ac \).

\textbf{Examples}

1. Find the solutions for the equation \( 8x^2 + 8x + 1 = 0 \).

   \textit{Step 1.} Express the equation in standard form \( ax^2 + bx + c = 0 \).

   The equation is already in standard form.

   \textit{Step 2.} Determine whether the quadratic can be readily factored.

   Try to factor the quadratic. After a few attempts, you’ll find that this quadratic is not factorable.
Skill IC8  

Find solutions to quadratic equations

Step 3. If the quadratic cannot be readily factored, substitute the numerical values for $a$, $b$, and $c$ into the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Simplify this expression.

The coefficients are $a = 8$, $b = 8$, and $c = 1$. Substitute these values into the quadratic formula as follows: $x = \frac{-8 \pm \sqrt{8^2 - 4(8)(1)}}{2(8)}$. Simplify.

\[
\frac{-8 \pm \sqrt{64 - 32}}{16} = \frac{-8 \pm \sqrt{32}}{16} = \frac{-8 \pm \sqrt{16} \times 2}{16} = \frac{-8 \pm 4\sqrt{2}}{16}
\]

Since each of the three terms in $\frac{-8 \pm 4\sqrt{2}}{16}$ has a common factor of 4, reduce the expression by dividing each term by 4 to obtain $x = \frac{-2 \pm \sqrt{2}}{4}$.

The solutions are $x = \frac{-2 + \sqrt{2}}{4}$ and $x = \frac{-2 - \sqrt{2}}{4}$.

2. Find the real roots of the equation $2x^2 + x = 3$.

Step 1. Express the equation in standard form $ax^2 + bx + c = 0$.

Subtract 3 from each side to obtain $2x^2 + x - 3 = 0$.

Step 2. Determine whether the quadratic can be readily factored.

The quadratic $2x^2 + x - 3 = 0$ will factor as $(2x + 3)(x - 1) = 0$.

Step 3. If the quadratic can be readily factored, factor the quadratic. Set each factor equal to zero and solve each equation to find the solution(s) or root(s).

\[
\begin{align*}
2x + 3 &= 0 \\
2x &= -3 \\
x &= \frac{-3}{2}
\end{align*}
\]

and

\[
\begin{align*}
x - 1 &= 0 \\
x - 1 + 1 &= 0 + 1 \\
x &= 1
\end{align*}
\]

The roots are $\frac{-3}{2}$ and 1.
Skill IC8

Find solutions to quadratic equations

Reminder: The quadratic formula could have been used to produce the same solutions. Substitute \(a = 2\), \(b = 1\), and \(c = -3\) into the quadratic formula.

\[
x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-3)}}{2(2)} = \frac{-1 \pm \sqrt{1 + 24}}{4} = \frac{-1 \pm \sqrt{25}}{4} = \frac{-1 \pm 5}{4}
\]

\[
\frac{-1 + 5}{4} = \frac{4}{4} = 1 \quad \text{and} \quad \frac{-1 - 5}{4} = \frac{-6}{4} = -\frac{3}{2}
\]

So the roots are \(-\frac{3}{2}\) and 1.
SOLVE A SYSTEM OF TWO LINEAR EQUATIONS IN TWO UNKNOWNS

A linear equation in two variables can be written in standard form $ax + by = c$, where $a$, $b$, and $c$ are real numbers with either $a \neq 0$ or $b \neq 0$. Each linear equation has infinitely many solutions of the form $(x, y)$ that satisfy the equation. That is, when these values are substituted into the equation, the resulting statement is true. For example, find some of the solutions for the linear equation $x + y = 5$.

1 + 4 = 5, so (1, 4) satisfies the linear equation.
2 + 3 = 5, so (2, 3) satisfies the linear equation.
5 + 0 = 5, so (5, 0) satisfies the linear equation.

This arbitrary process of finding pairs that satisfy one linear equation is endless since there are infinitely many such ordered pairs. But, a system of two linear equations will require that an ordered pair satisfy both equations simultaneously. There are three possibilities that arise when solving a system of two linear equations.

A. The system has one and only one ordered pair $(x, y)$.
B. The system has no solutions (empty set).
C. The system has infinitely many solutions, or ordered pairs.

Solving a System of Linear Equations, Using the Addition Method

The objective of the addition, or elimination, method is to eliminate one of the variables and solve for the one remaining variable. When one value has been determined, substitute it into either one of the original equations to find the other value of the pair. The steps of this method are as follows:

Step 1. Write each equation in standard form $ax + by = c$. Determine which coefficients of the $x$ and $y$ terms will be easiest to make into opposite values (additive inverses).

Step 2. If necessary, multiply one or both of the equations by a nonzero constant to make the chosen set of coefficients opposites. This creates a new system of equations that are multiples of the original equations.

Step 3. Add these two new equations. The result is a single equation with, at most, one variable. Solve for this remaining variable.

Step 4. Substitute this value into either of the original equations and solve for the other variable.
Skill IC9  *Solve a system of two linear equations in two unknowns*

**Examples**

1. Solve the system of equations:  
   \[ 2x - y = 4 \]
   \[ 3x - 2y = 2 \]

   **Step 1.** The equations are in standard form. Determine which coefficients of the \( x \) and \( y \) terms will be easiest to make into opposite values (additive inverses).
   
   The coefficients of the \( y \) terms can be made into opposite values by multiplying the first equation by \(-2\).

   **Step 2.** If necessary, multiply one or both of the equations by a nonzero constant to make the chosen sets of coefficients opposites.
   
   Multiply the first equation by \(-2\) and copy the second equation.
   
   \[ -4x + 2y = -8 \]
   \[ 3x - 2y = 2 \]

   **Step 3.** Add these two new equations and solve for the remaining variable.
   
   \[ -x = -6 \]
   \[ x = 6 \]

   **Step 4.** Substitute this value into either of the original equations and solve for the other variable.
   
   \[ 2(6) - y = 4 \]
   \[ -y = -8 \]
   \[ y = 8 \]

   The apparent solution \((6, 8)\) must satisfy the other equation also, so this should be checked. \(3(6) - 2(8) = 18 - 16 = 2\), which does satisfy the equation.

   Therefore, the solution to the system of equations is the ordered pair \((6, 8)\).

The previous exercise is an example of a system that has only one pair of values that satisfies both equations. But, as discussed earlier, there are two other possibilities: the empty set and infinitely many solutions. The following examples will illustrate how these possibilities arise and how to solve such systems.

2. Solve the system of equations:  
   \[ 2x + 3y = 2 \]
   \[ 4x + 6y = 3 \]

   **Step 1.** The equations are in standard form. Both sets of coefficients require only one multiplication to make opposite values, so one choice is to make the coefficients of the \( x \) terms into opposite values.
Skill IC9  Solve a system of two linear equations in two unknowns

Step 2. If necessary, multiply one or both of the equations by a nonzero constant to make the chosen sets of coefficients opposites.

Multiply the first equation by \(-2\) and copy the second equation.

\(-4x - 6y = -4\)
\(4x + 6y = 3\)

Step 3. Add these two new equations and note that there is no remaining variable.

\(0 + 0 = -1\)
\(0 = -1\)

Step 4. Substitute this value into either of the original equations and solve for the other variable.

The intention was to eliminate only one variable, but the multiplication created two sets of opposite coefficients and both variables were eliminated. This leaves no variable for which to solve. In fact, the resulting statement is false (a numerical contradiction).

Reminder: When both variables are eliminated and the resulting statement is false, the solution is the empty set (i.e., the system has no solutions).

3. Solve the system of equations:

\[ 2x - y = 2 \]
\[ 4x - 2y = 4 \]

Step 1. The equations are in standard form. It is easy to see that the coefficients of the \(x\) terms can be made into opposites with one multiplication.

Step 2. If necessary, multiply one or both of the equations by a nonzero constant to make the chosen sets of coefficients opposites.

Multiply the first equation by \(-2\) and copy the second equation.

\(-4x + 2y = -4\)
\(4x - 2y = 4\)

Step 3. Add these two equations.

\(0 + 0 = 0\)
\(0 = 0\)

Step 4. Substitute this value into either of the original equations and solve for the other variable.

Again, both variables were eliminated, but this time the resulting statement is true. When both variables are eliminated and the resulting statement is true, the solution is infinitely many ordered pairs \((x, y)\).
Skill IC9  Solve a system of two linear equations in two unknowns

Reminder: Infinitely many ordered pairs does not mean that every ordered pair satisfies the system of equations. It means that any ordered pair that satisfies one of the equations will also satisfy the other. Because it is impossible to list all of these ordered pairs, the solution is often written in the following set-builder notation: \(\{(x, y) \mid 2x - y = 2\}\) or \(\{(x, y) \mid y = 2x - 2\}\).

4. Find the solution set for the system of linear equations:
   \[\begin{align*}
   5x - 3y &= -2 \\
   2x + 2y &= -8
   \end{align*}\]

   **Step 1.** The equations are in standard form. Both sets of coefficients require two multiplications to make opposite values, so one choice is to make the coefficients of the \(x\) terms into opposite values.

   **Step 2.** If necessary, multiply one or both of the equations by a nonzero constant to make the chosen sets of coefficients opposites.

   Multiply the first equation by \(-2\) and the second equation by \(5\).
   \[-10x + 6y = 4\]
   \[10x + 10y = -40\]

   **Step 3.** Add these two new equations and solve for the other variable.

   \[16y = -36\]
   \[y = -\frac{36}{16} = -\frac{9}{4}\]

   **Step 4.** Substitute this value into either of the original equations and solve for the other variable.

   \[2x + 2\left(-\frac{9}{4}\right) = -8\]
   \[2x - \frac{9}{2} = -8\]
   \[2x = -8 + \frac{9}{2}\]
   \[x = -4 + \frac{9}{4}\]
   \[x = -\frac{7}{4}\]

   Therefore, the solution set is \(\left\{\left(-\frac{7}{4}, -\frac{9}{4}\right)\right\}\).
**Skill IIC4** Identify specified regions of the coordinate plane

**IDENTIFY SPECIFIED REGIONS OF THE COORDINATE PLANE**

A **coordinate plane** is a plane in which a system of ordered pairs of numbers are graphed in relation to two axes (horizontal and vertical) that intersect at their zero point. A linear inequality in two variables \(x\) and \(y\) can be expressed in one of the following forms:

\[
ax + by \leq c \quad ax + by \geq c \\
ax + by < c \quad ax + by > c
\]

where \(a\), \(b\), and \(c\) are real numbers with \(a \neq 0\) and/or \(b \neq 0\). The objective is to identify the set of points \((x, y)\) in the coordinate plane that satisfy the condition(s) stated by these inequalities. This set of points will be expressed graphically as a shaded region. This process involves two steps:

**Step 1.** Sketch the **boundary line**. This line represents the points that satisfy the equation \(ax + by = c\). If the inequality is < or >, this line will be dashed. This indicates that the line is the edge of the region, but that the points on the line do not satisfy the inequality. If the inequality is \(\leq\) or \(\geq\), the boundary line will be solid. This indicates that the points on the line do satisfy the inequality.

**Step 2.** Shade the region satisfying the inequality. This shaded region represents all points in the coordinate plane that satisfy the strict inequality condition(s).
**Skill IIC4**  
*Identify specified regions of the coordinate plane*

**Sketch the Boundary Line**

To sketch the boundary line, ignore the inequality symbol and concentrate on the linear equation $ax + by = c$. If $a$ and $b$ are both nonzero coefficients, find two points on this line by arbitrarily selecting two $x$-values and substituting them into the equation to find the corresponding $y$-values. Plot these points and sketch the line through them.

While any two points will suffice, if $a$, $b$, and $c$ are all nonzero, such as $2x - y = 6$, it is convenient to find the intercepts of the line, that is, the points where the line crosses the coordinate axes.

- To find the $x$-intercept, substitute 0 for $y$ and solve for $x$.
- To find the $y$-intercept, substitute 0 for $x$ and solve for $y$.

Using these intercepts to sketch the line is called the “intercept method.” If $c = 0$ while $a$ and $b$ are nonzero, such as $x + 3y = 0$, the line will pass through the origin. In this case, the intercept method will produce only one point, $(0, 0)$. A second point can be found by arbitrarily selecting a nonzero $x$-value and substituting for $x$ to find the corresponding $y$-value.

If the equation is missing either the $x$ term or the $y$ term, such as $x = 3$, it is one of the following special cases:

**Vertical Line:** If the $y$ term is missing, isolate the $x$ term to produce an equation of the form $x = k$, where $k$ is a constant. This line will be vertical, passing through the $x$-axis at $(k, 0)$. 

---

201
**Skill IIC4**  
*Identify specified regions of the coordinate plane*

**Horizontal Line:** If the $x$ term is missing, isolate the $y$ term to produce an equation of the form $y = k$, where $k$ is a constant. This line will be horizontal, passing through the $y$-axis at $(0, k)$.

![Horizontal Line Diagram]

**Shade the Appropriate Region**

The boundary line divides the coordinate plane into two regions. Choose any point in the plane that is not on the boundary line to serve as a test point. Substitute the coordinates of this point into the original inequality. The resulting numerical statement will either be true or false. If it is true, shade the region containing the test point. If it is false, shade the region not containing the test point.

**Examples**

1. Shade the region of the coordinate plane satisfying the inequality $2x - y \leq 6$.

   **Step 1.** Sketch the boundary line $2x - y = 6$ as a solid line. Since $a$, $b$, and $c$ are all non-zero, the intercept method can be used.

   If $x = 0$, then $2(0) - y = 6$
   
   $-y = 6$
   
   $y = -6$

   point $(0, -6)$

   If $y = 0$, then $2x - 0 = 6$

   $2x = 6$

   $x = 3$

   point $(3, 0)$

   **Step 2.** Choose a test point not on the boundary line, such as $(0, 0)$. Substitute 0 for $x$ and 0 for $y$ in the original inequality.

   $2(0) - 0 \leq 6$

   $0 \leq 6$

   Since this statement is true, shade the region containing $(0, 0)$.

![Test Point Diagram]
Skill IIC4  Identify specified regions of the coordinate plane

2. Shade the region of the coordinate plane satisfying the inequality \( y > 2 \).

   \textit{Step 1.} Sketch the boundary line \( y = 2 \) as a dashed line. Since the \( x \) term is missing, the line is horizontal, passing through the \( y \)-axis at \((0, 2)\).

   \textit{Step 2.} Choose a test point such as \((0, 0)\). Substituting the \( y \)-coordinate 0 into the original inequality produces the statement \( 0 > 2 \). Since this statement is false, shade the region \textit{not} containing \((0, 0)\).

   

\[ \text{And and Or} \]

When two inequalities are connected by the word \textit{and}, graph the inequalities separately and shade the region whose points satisfy both inequalities simultaneously. This region will be the \textit{intersection} (overlap) of the two regions.

\textbf{Examples}

3. Shade the region of the coordinate plane satisfying the condition that \( x + 3y \geq 0 \) and \( x < 3 \).

   \textit{Step 1.} Sketch the solid boundary line \( x + 3y = 0 \). Since \( a \) and \( b \) are nonzero while \( c = 0 \), the line will pass through \((0, 0)\). Find a second point by arbitrarily selecting a nonzero \( x \)-value, such as \( x = 3 \). Substitute 3 for \( x \):

   \[
   \begin{align*}
   3 + 3y &= 0 \\
   3y &= -3 \\
   y &= -1
   \end{align*}
   \]

   The boundary line will pass through \((0, 0)\) and \((3, -1)\).
Skill IIC4

*Identify specified regions of the coordinate plane*

**Step 2.** Choose a test point such as $(1, 1)$. Substitute 1 for $x$ and 1 for $y$ in the original inequality:

$$1 + 3(1) \geq 0$$
$$4 \geq 0$$

Since this statement is true, shade the region containing the point $(1, 1)$.

Repeat Steps 1 and 2 for the second inequality.

**Step 1.** Sketch the dashed boundary line $x = 3$. Since the $y$ term is missing, this is a vertical line through $(3, 0)$.

**Step 2.** Choose a test point such as $(0, 0)$. Substitute 0 for $x$ in the original inequality: $0 < 3$. Since this statement is true, shade the region containing the point $(0, 0)$.

The intersection (overlap) region satisfies both inequalities simultaneously.

When two inequalities are connected by the word *or*, shade the region whose points satisfy at least one of the inequalities, that is, the points that satisfy either the first inequality or the second inequality, or both. This desired region is the *union* of the two regions.
Skill IIIC4  *Identify specified regions of the coordinate plane*

4. Shade the region of the coordinate plane satisfying the condition that $x \geq 2$ or $y > 0$.

The individual regions are shown below.

When one sketch is superimposed over the other, the union consists of the set of points that satisfy either $x \geq 2$ or $y > 0$ (or both).
SOLVE PROBLEMS INVOLVING THE STRUCTURE AND LOGIC OF ALGEBRA

When an algebraic relationship between two or more quantities is described in words, it is desirable to express the relationship symbolically as an equation or inequality. To construct this equation or inequality, choose a variable to represent one quality, and express all other quantities in terms of this variable.

Examples

1. If \( x \) represents the smallest of three consecutive integers, find an algebraic statement that is equivalent to the following verbal statement:

   "The sum of three consecutive integers is always 3 more than three times the smallest integer."

   For the first part of the statement, "the sum of three consecutive integers," let \( x = \) the smallest of three consecutive integers. Then \( x + 1 = \) the middle integer, and \( x + 2 = \) the largest integer. The phrase would be expressed as \( x + (x + 1) + (x + 2) \), or more simply \( x + x + 1 + x + 2 \).

   The second part of the statement, "3 more than three times the smallest integer," would be expressed as \( 3x + 3 \). Therefore, the complete statement is expressed by the equation \( x + x + 1 + x + 2 = 3x + 3 \).

In example 1, note that the phrase "3 more than" means that 3 had to be added to \( 3x \). Certain key phrases occur often in these problems. The following is a list of some of the more common phrases, along with their translation.

- A number increased by 7 \( = x + 7 \)
- 7 more than a number \( = x + 7 \)

- A number decreased by 7 \( = x - 7 \)
- 7 less than a number \( = x - 7 \)

- The product of 2 and a number \( = 2x \)
- 2 times a number \( = 2x \)
- A number doubled \( = 2x \)

- The reciprocal of a number \( = \frac{1}{x} \)

- The square of a number \( = x^2 \)
Skill IVC2  Solve problems involving the structure and logic of algebra

2. Write the equation needed to find the number \( x \) if the number decreased by its reciprocal is \( \frac{24}{5} \).

Let \( x \) be the number. Then \( \frac{1}{x} \) is the reciprocal of \( x \). The phrase "the number decreased by its reciprocal" means subtract the reciprocal from the number itself.

That is, \( x - \frac{1}{x} \). Therefore, the equation is \( x - \frac{1}{x} = \frac{24}{5} \).

Another type of problem involving a standardized relationship is the two-digit integer problem. Recall that the base-ten system is made up of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The next two examples illustrate two-digit integer problems.

Examples

3. The sum of the digits of a two-digit number is 11. If the tens digit is 1 less than 5 times the units digit, what equation could be used to find \( x \), the units digit?

Let \( x \) be the units digit. Then 5\( x \) – 1 = the tens digit. The phrase "the sum of the digits of a two-digit number is 11" means that:

the units digit + the tens digit = 11.

Therefore, the equation can be expressed as \( x + 5x - 1 = 11 \).

4. The units digit of a two-digit number is 4 more than the tens digit. The number itself is 4 less than 5 times the units digit. Find an equation that could be used to find the tens digit, \( x \).

Let \( x \) be the tens digit. The first sentence indicates that the units digit = \( x + 4 \). Expressing the number itself involves the following property of the base-ten system:

a two-digit number = 10(tens digit) + units digit

For instance, 53 = 10(5) + 3.

Therefore, the number itself is expressed by 10\( x \) + (\( x + 4 \)). The phrase "4 less than 5 times the units digit" can be expressed as 5(\( x + 4 \)) – 4.

Therefore, the equation is 10\( x \) + (\( x + 4 \)) = 5(\( x + 4 \)) – 4.
Skill IVC2  Solve problems involving the structure and logic of algebra

5. For a two-digit number, let \( u \) = the units digit and \( t \) = the tens digit. Find an equation that is equivalent to “a two-digit number is 2 more than 4 times the units digit.”

Let \( u \) = units digit.
Let \( t \) = tens digit.
\( 10t + u \) = the “two-digit number.”
\( 4u + 2 \) = “2 more than 4 times the units digit.”

\( 10t + u = 4u + 2 \)
IDENTIFY STATEMENTS OF PROPORTIONALITY AND VARIATION

Consider two variable quantities \( x \) and \( y \). If an increase in \( x \) produces a proportional increase in \( y \), the quantities \( x \) and \( y \) vary directly with one another. If an increase in \( x \) produces a proportional decrease in \( y \), the quantities \( x \) and \( y \) vary inversely with one another. To solve problems involving direct or inverse variation, it is often helpful to express variation as an equation that utilizes the “proportional” relationship between the variables.

Direct Variation

If two quantities \( x \) and \( y \) are related by the equation \( \frac{y}{x} = k \), or equivalently, \( y = kx \), where \( k \) is the constant of proportionality, this means that \( y \) varies directly with \( x \). Since the ratio of \( y \) to \( x \) is constant, then \( y \) is proportional to \( x \). When the given information in a problem indicates that two quantities are directly proportional, there are two methods for expressing this relationship algebraically:

**Method 1:** Identify the variations in \( x \) and \( y \) as ratios and set up the proportion: \( \frac{x_1}{x_2} = \frac{y_1}{y_2} \).

**Method 2:** Identify the factors \( x \) and \( y \) whose ratio is constant, calculate this constant of proportionality \( k \), and express the given condition as the equation: \( y = kx \).

Example

1. Suppose 3 equally paid men earn a total of $55.50 in wages for an hour’s work. Let \( x \) represent the number of equally paid men that could be paid at the same hourly wage with a total of $277.50 for an hour’s work. Write an equation that could be used to find \( x \) and solve for \( x \).

   Method 1: The number of men varies from \( x_1 = 3 \) to an unknown \( x \) while the total wage varies from \( y_1 = 55.50 \) to \( y_2 = 277.50 \). Since the hourly wage is constant, these variations (ratios) should be proportional.

   \[
   \frac{x_1}{x} = \frac{y_1}{y_2} \Rightarrow \frac{3}{x} = \frac{55.50}{277.50} \Rightarrow (55.5)x = (3)(277.5) \Rightarrow x = 15
   \]

   Method 2: The hourly wage is the constant of proportionality, \( k \). Since \( y = kx \), use the fact that 3 men are paid a total of $55.50 to find \( k \).

   \[
   \frac{55.50}{3} = k \Rightarrow k = 18.50
   \]

   Since total wage = (hourly wage)(number of men), we have $277.50 = $18.50x. Therefore, \( x = 15 \).
Skill IIC3

**Identify statements of proportionality and variation**

In example 1, note that either equation produces the solution $x = 15 \text{ men}$. Although the two methods produce equations that look different, they really are equivalent equations.

**Inverse Variation**

If the product of the two quantities remains constant, they are related by the equation: $xy = k$, where $k$ is the constant of proportionality and the quantity $y$ varies inversely with $x$.

When the given information in a problem indicates that two quantities are inversely proportional, there are two methods for expressing this relationship algebraically.

**Method 1:**

**Step 1.** Identify the variations in $x$ and $y$ as ratios.

**Step 2.** Set up the proportion $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

**Method 2:**

**Step 1.** Identify the factors $x$ and $y$ with products that will remain constant.

**Step 2.** Express the given condition as the equation $xy = k$.

**Example**

2. Traveling at the 55 mph speed limit, it takes 6 hours to drive from Gainesville, FL to Atlanta, GA. Let $t$ represent the time required for the same trip traveling at 60 mph. Write a statement representing this condition.

**Method 1:**

**Step 1.** Identify the variations in $x$ and $y$ as ratios.

$r_1 = 55 \text{ mph}, r_2 = 60 \text{ mph}, t_1 = 6 \text{ hours}, t_2 = ?$

**Step 2.** Set up the proportion $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

$$\frac{r_1}{r_2} = \frac{t_1}{t_2} \quad \frac{55}{60} = \frac{6}{t_2}$$

Other equivalent forms of this property are $\frac{60}{55} = \frac{6}{t_2}$, $\frac{t_1}{6} = \frac{55}{60}$, and $\frac{55}{t_2} = \frac{60}{6}$.

Note that the cross product for each property is the same, $60t_2 = 330$. 

210
Skill II.C3  Identify statements of proportionality and variation

Method 2:
Step 1. Identify the factors $x$ and $y$ with products that will remain constant.
$r_1 = 55 \text{ mph}, \ r_2 = 60 \text{ mph}, \ t_1 = 6 \text{ hours}, \ \text{distance} = k$

Step 2. Express the given condition as the equation $xy = k$.
So if $xy = k$, then $k = (55)(6) = 330 \text{ miles}$.
The inverse is $60(t_2) = 330$. 
SOLVE ALGEBRAIC WORD PROBLEMS WITH VARIABLES

On the surface, real-world word problems may seem totally different from one another. But there are a few steps that should be part of the algebraic solution to any word problem.

Step 1. Read the problem carefully.

Step 2. Organize the problem on paper.

A. If the problem gives some type of formula,
   - identify all the known quantities and the unknown quantity, and
   - substitute the known quantities into the formula and solve for the unknown.

B. If no formula is given,
   - find the sentence or phrase that actually asks the question, and select a variable to represent the unknown,
   - express any other unknown quantities in terms of this variable using key phrases in the problem,
   - rule out any irrelevant information,
   - identify a relationship between the various quantities and express this relationship as an equation, and then
   - solve the equation.

Step 3. Check the solution.

A. If a formula is given, check to be sure that the solution satisfies the formula when substituted for the unknown.

B. If no formula is given, check to make sure that the solution satisfies the conditions of the problem.
Skill IVC1  Solve algebraic word problems with variables

Examples

1. Suppose an object is dropped from an initial height $S_0$ and falls vertically to the ground while subjected only to the force of gravity $g$. The formula $S = S_0 - \frac{1}{2} gt^2$ gives the height $S$ of the object at time $t$. Find the initial height $S_0$ if the height of the object at 3 seconds is 56 feet, and the force of gravity is $32 \frac{\text{ft.}}{\text{sec.}^2}$.

*Step 1.* Read the problem carefully. Note that a formula has been given.

*Step 2.* Organize the problem on paper.

- Identify the known and unknown quantities.

  The given values are $t = 3$ seconds, $S = 56$ feet, and $g = 32 \frac{\text{ft.}}{\text{sec.}^2}$.

- Substitute these values into the formula $S = S_0 - \frac{1}{2} gt^2$, and solve for the unknown.

  \[
  56 = S_0 - \frac{1}{2} (32)(3)^2
  \]

  \[
  56 = S_0 - 144
  \]

  \[
  S_0 = 200 \text{ feet}
  \]

*Step 3.* Check the solution. Substitute 200 for $S_0$.

  \[
  56 = 200 - \frac{1}{2} (32)(3)^2
  \]

  \[
  56 = 200 - 144
  \]

  \[
  56 = 56 \text{ (true)}
  \]

Therefore, the initial height is 200 feet.

2. According to the ideal gas law of physics, when the temperature of a gas is held constant, the volume, $V$, of the gas varies inversely with the pressure, $P$. If the pressure is $\frac{600 \text{ lb.}}{\text{in.}^2}$ when the volume is 50 in.$^3$, what is the pressure when the volume is 30 in.$^3$?
Skill IVC1  

*Solve algebraic word problems with variables*

*Step 1.* Read the problem carefully. Note that no formula is given.

*Step 2.* Organize the problem on paper.

- The question asked is, “What is the pressure when the volume is 30 in.³?”
  
  Let $P_2 =$ pressure when the volume is 30 in.³.

- Since volume, $V$, varies inversely with pressure, $P$, the relationship between quantities can be expressed as $\frac{V_1}{V_2} = \frac{P_2}{P_1}$.

- Substitute and solve the equation.

  \[
  \frac{V_1}{V_2} = \frac{P_2}{P_1} \quad \frac{50}{30} = \frac{P_2}{600} \quad 30P_2 = 30,000 \quad P_2 = 1000 \text{ lb./in.}^2
  \]

*Step 3.* Check the solution. If $P$ and $V$ vary inversely, then the pressure, $P$, should be larger than 600 when volume decreases from 50 to 30. Our solution, 1000, satisfies this condition. Substitute 1000 for $P_2$ in the proportion.

\[
\frac{50}{30} = \frac{1000}{600} \quad \frac{5}{3} = \frac{5}{3} \quad \text{(true)}
\]

Therefore, when volume is 30 in.³, pressure is 1000 lb./in.².

3. Mr. Henry is a car salesman. He receives a weekly salary of $200 plus 2% commission on each sale he makes. If the commission on Mr. Henry’s last sale was $300, what was the amount of the sale?

*Step 1.* Read the problem carefully. Note that no formula is given.

*Step 2.* Organize the problem on paper.

- The question asked is, “What was the amount of sale?”
  
  Let $x =$ the amount of Mr. Henry’s last sale.

- $0.02x =$ the commission of his last sale, $C$. 

214
Skill IVC1  Solve algebraic word problems with variables

- His weekly base salary of $200 is irrelevant information.
- Since his commission was $300. The relationship of the quantities can be expressed as $0.02x = C$.
- Substitute and solve the equation.

\[
0.02x = 300 \\
x = \frac{300}{0.02} = 15,000
\]

Step 3. Check the solution.

2% of $15,000 is $300, so the amount of the sale was $15,000.
# ALGEBRA

## PRACTICE PROBLEMS

### SKILLS

<table>
<thead>
<tr>
<th></th>
<th>PROBLEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC1</td>
<td>Add, subtract, multiply, and divide real numbers</td>
</tr>
<tr>
<td>IC2</td>
<td>Apply the order of operations agreement</td>
</tr>
<tr>
<td>IC1I</td>
<td>Use properties of operations</td>
</tr>
<tr>
<td>IC3</td>
<td>Use scientific notation</td>
</tr>
<tr>
<td>IC2II</td>
<td>Determine if a number is a solution to an equation or inequality</td>
</tr>
<tr>
<td>IC2III</td>
<td>Use properties to identify equivalent equations and inequalities</td>
</tr>
<tr>
<td>IC4</td>
<td>Solve linear equations and inequalities</td>
</tr>
<tr>
<td>IC5</td>
<td>Use algebraic formulas</td>
</tr>
<tr>
<td>IC6</td>
<td>Find values of functions</td>
</tr>
<tr>
<td>IC7</td>
<td>Find factors of quadratic expressions</td>
</tr>
<tr>
<td>IC8</td>
<td>Find solutions to quadratic equations</td>
</tr>
<tr>
<td>IC9</td>
<td>Solve a system of two linear equations in two unknowns</td>
</tr>
<tr>
<td>IC4II</td>
<td>Identify specified regions of the coordinate plane</td>
</tr>
<tr>
<td>IVC2</td>
<td>Solve problems involving the structure and logic of algebra</td>
</tr>
<tr>
<td>IIC3</td>
<td>Identify statements of proportionality and variation</td>
</tr>
<tr>
<td>IVC1</td>
<td>Solve algebraic word problems with variables</td>
</tr>
</tbody>
</table>

217
### ADD, SUBTRACT, MULTIPLY, AND DIVIDE REAL NUMBERS

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $5\sqrt{12} + 2\sqrt{3} - \sqrt{27} =$</td>
<td>4. $\sqrt{5} \times \sqrt{8} =$</td>
<td></td>
</tr>
<tr>
<td>A. $7\sqrt{-12}$</td>
<td>A. $2\sqrt{10}$</td>
<td></td>
</tr>
<tr>
<td>B. $6\sqrt{3}$</td>
<td>B. $4\sqrt{10}$</td>
<td></td>
</tr>
<tr>
<td>C. $9\sqrt{3}$</td>
<td>C. $\sqrt{13}$</td>
<td></td>
</tr>
<tr>
<td>D. $13\sqrt{3}$</td>
<td>D. $25 \times 64$</td>
<td></td>
</tr>
</tbody>
</table>

| 2. $9\pi + 11\pi - 1 =$ | 5. $\frac{\sqrt{40}}{\sqrt{2}} =$ |
| A. $19$ | A. $2\sqrt{5}$ |
| B. $19\pi$ | B. $4\sqrt{5}$ |
| C. $20\pi - 1$ | C. $2\sqrt{10}$ |
| D. $20\pi^2 - 1$ | D. $\sqrt{40}$ |

| 3. $2\pi + \sqrt{9}\pi =$ | 6. $\frac{3}{\sqrt{2}} =$ |
| A. $2 + \sqrt{9}\pi$ | A. $\frac{\sqrt{3}}{2}$ |
| B. $5\pi$ | B. $\frac{3\sqrt{2}}{4}$ |
| C. $11\pi$ | C. $\frac{3\sqrt{2}}{2}$ |
| D. $83\pi$ | D. $3\sqrt{2}$ |
7. \[ \frac{2\pi \times 3\pi^2}{3\pi} \]

A. \[ \frac{5\pi^2}{3} \]

B. \[ 2\pi \]

C. \[ 2\pi^2 \]

D. \[ 3\pi^2 \]
APPLY THE ORDER OF OPERATIONS AGREEMENT

8. \( 2^3 + 3(x - 2) = \)
   
   A. \( 3x \)
   B. \( 3x + 2 \)
   C. \( 3x + 6 \)
   D. \( 5x \)

9. \( \frac{3}{4} + \frac{1}{3} + \frac{4}{3} = \)
   
   A. \( \frac{1}{3} \)
   B. \( \frac{3}{7} \)
   C. \( \frac{1}{2} \)
   D. 1
10. Choose the expression equivalent to the following:

\[ 8y + 4x \]

A. \(12xy\)
B. \(8(y + 4x)\)
C. \(4(2y + x)\)
D. \(2y + x\)

11. Choose the statement that is NOT true for all real numbers.

A. for \(a \neq 0\), \(a \times 0 = a\)
B. \(a + (-a) = 0\)
C. \(2(ab) = (2a)b\)
D. \(a(b + 1) = ab + a\)
12. $0.00025 \div 5,000,000 =$

   A. $5.0 \times 10^{-11}$
   B. $5.0 \times 10^{-10}$
   C. $1.25 \times 10^3$
   D. $5.0 \times 10^2$

13. $(6.3 \times 10^4) \times (1.2 \times 10^{-5}) =$

   A. -7.56
   B. 0.0756
   C. 0.756
   D. 75.6
14. For each of the statements below, determine whether \( x = -1 \) is a solution.

i. \( |x - 3| > 3 \)

ii. \( (2x - 1)(x + 4) = -9 \)

iii. \( 2 - 4x = x - 5 \)

A. i, ii, and iii
B. i and ii only
C. i only
D. ii only

15. For each of the statements below, determine whether \( x = \frac{1}{2} \) is a solution.

i. \( 6x \leq 4x^2 + 2 \)

ii. \( 10x + 1 = 6(4x - 3) \)

iii. \( |x - 1| = x \)

A. i, ii, and iii
B. i and iii only
C. i only
D. iii only
### Algebra Practice Problems

**Skill HIC2**

**USE PROPERTIES TO IDENTIFY EQUIVALENT EQUATIONS AND INEQUALITIES**

<table>
<thead>
<tr>
<th>16. Choose the equation that is equivalent to the following: $5 - 3x = x + 1$</th>
<th>19. Given that $y &lt; 0$, choose the inequality that is equivalent to the following: $1 &lt; \frac{x}{y} &lt; 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $-3x = x + 4$</td>
<td>A. $1 &gt; x &gt; 5$</td>
</tr>
<tr>
<td>B. $3x = x - 4$</td>
<td>B. $y &gt; x &gt; 5y$</td>
</tr>
<tr>
<td>C. $-3x = -5x + 1$</td>
<td>C. $y &lt; x &lt; 5y$</td>
</tr>
<tr>
<td>D. $-3x = x - 4$</td>
<td>D. $5y &gt; x &gt; y$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>17. Choose the inequality that is equivalent to the following: $x^2 - 6 &gt; x$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $x^2 - x - 6 &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>B. $x^2 &lt; x - 6$</td>
<td></td>
</tr>
<tr>
<td>C. $x^2 - x - 6 &lt; 0$</td>
<td></td>
</tr>
<tr>
<td>D. $x^2 + x - 6 &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>18. Choose the equation that is equivalent to the following: $\frac{x}{4} - 5 = 2x$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $x - 5 = 8x$</td>
<td></td>
</tr>
<tr>
<td>B. $x - 20 = 2x$</td>
<td></td>
</tr>
<tr>
<td>C. $x - 20 = 8x$</td>
<td></td>
</tr>
<tr>
<td>D. $4x - 20 = 8x$</td>
<td></td>
</tr>
</tbody>
</table>
## SOLVE LINEAR EQUATIONS AND INEQUALITIES

20. If $3x - 1 = 2(5 - 4x) + 3$, then

<table>
<thead>
<tr>
<th>Option</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>$x = \frac{11}{14}$</td>
</tr>
<tr>
<td>B.</td>
<td>$x = \frac{14}{11}$</td>
</tr>
<tr>
<td>C.</td>
<td>$x = \frac{18}{11}$</td>
</tr>
<tr>
<td>D.</td>
<td>$x = 2$</td>
</tr>
</tbody>
</table>

21. If $4 - 4m < 16$, then

<table>
<thead>
<tr>
<th>Option</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>$m &gt; -3$</td>
</tr>
<tr>
<td>B.</td>
<td>$m &lt; -3$</td>
</tr>
<tr>
<td>C.</td>
<td>$m &gt; 4$</td>
</tr>
<tr>
<td>D.</td>
<td>$m &lt; 16$</td>
</tr>
</tbody>
</table>

22. If $4x - (3 - x) = 6(x - 3) + 10$, then

<table>
<thead>
<tr>
<th>Option</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>$x = -5$</td>
</tr>
<tr>
<td>B.</td>
<td>$x = -\frac{10}{3}$</td>
</tr>
<tr>
<td>C.</td>
<td>$x = \frac{5}{3}$</td>
</tr>
<tr>
<td>D.</td>
<td>$x = 5$</td>
</tr>
</tbody>
</table>
23. The formula for converting a Fahrenheit temperature to Celsius is

\[ C = \frac{5}{9} (F - 32^\circ). \]

What is the temperature on the Celsius scale when the Fahrenheit temperature is 77^\circ?

A. 10.8^\circ
B. 25^\circ
C. 81^\circ
D. 198^\circ

24. Given the formula \( d = rt \) (where \( d \) = distance, \( r \) = rate, and \( t \) = time), calculate the time required for a vehicle to travel 275 miles at a rate of 50 miles per hour.

A. 5.25 hours
B. 5.5 hours
C. 52 hours
D. 13750 hours

25. Given the formula \( y = 2xt - r \), solve for \( t \).

A. \( t = \frac{y - r}{2x} \)
B. \( t = y + r - 2x \)
C. \( t = \frac{y}{2x} + r \)
D. \( t = \frac{y + r}{2x} \)
### Algebra Practice Problems

#### Skill IC6

**FIND VALUES OF FUNCTIONS**

26. Find \( f(-3) \), given \( f(x) = 2x^2 - 3x - 1 \).

   - A. 2
   - B. 8
   - C. 20
   - D. 26

27. Given \( f(x) = x^3 - 2x^2 - x \), find \( f(-2) \).

   - A. -18
   - B. -14
   - C. -12
   - D. 2
<table>
<thead>
<tr>
<th></th>
<th>FIND FACTORS OF QUADRATIC EXPRESSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.</td>
<td>Which is a linear factor of the following expression?</td>
</tr>
<tr>
<td></td>
<td>$3x^2 - x - 4$</td>
</tr>
<tr>
<td></td>
<td>A. $x - 2$</td>
</tr>
<tr>
<td></td>
<td>B. $x + 2$</td>
</tr>
<tr>
<td></td>
<td>C. $3x - 4$</td>
</tr>
<tr>
<td></td>
<td>D. $3x + 2$</td>
</tr>
<tr>
<td>29.</td>
<td>Which is a linear factor of the following expression?</td>
</tr>
<tr>
<td></td>
<td>$6x^2 - x - 2$</td>
</tr>
<tr>
<td></td>
<td>A. $x - 2$</td>
</tr>
<tr>
<td></td>
<td>B. $2x - 1$</td>
</tr>
<tr>
<td></td>
<td>C. $3x - 2$</td>
</tr>
<tr>
<td></td>
<td>D. $3x + 1$</td>
</tr>
</tbody>
</table>
### FIND SOLUTIONS TO QUADRATIC EQUATIONS

30. Find the correct solutions to the equation $x^2 - 2x - 1 = 0$.

A. $1 - \sqrt{3}$ and $1 + \sqrt{2}$

B. $1 - \sqrt{2}$ and $1 + \sqrt{2}$

C. $1$ and $-2$

D. $1 - \sqrt{3}$ and $1 + \sqrt{3}$

31. Find the real roots of the equation $12x^2 + x = 6$.

A. $\frac{-3}{2}$ and $\frac{2}{3}$

B. $\frac{-3}{2}$ and $\frac{1}{3}$

C. $\frac{-3}{4}$ and $\frac{2}{3}$

D. $\frac{-1 + \sqrt{-287}}{24}$ and $\frac{-1 - \sqrt{-287}}{24}$
32. Choose the correct solution set for the system of linear equations.

\[ 5x - 3y = 25 \]
\[ x + 2y = -8 \]

A. \{(2, -5)\} 
B. \{(-8, 0)\} 
C. \{(2, 5)\} 
D. the empty set

34. Choose the correct solution set for the system of linear equations.

\[ 2x - 6y = 4 \]
\[ -x + 3y = -2 \]

A. \{(0, 0)\} 
B. \{(5, 1)\} 
C. the empty set 
D. \( \left\{ (x, y) \right\} \mid y = \frac{x-2}{3} \)

33. Choose the correct solution set for the system of linear equations.

\[ -15x + 9y = -6 \]
\[ 5x - 3y = 6 \]

A. \( \left\{ \left( \frac{6}{5}, 1 \right) \right\} \)
B. \( \left\{ \left( 0, -\frac{2}{3} \right) \right\} \)
C. \( \left\{ (x, y) \right\} \mid y = \frac{5x - 6}{3} \)
D. the empty set
35. Select the shaded region of the coordinate plane satisfying the condition \( x + 3y \geq 6 \).

A.  

B.  

C.  

D.  

36. Identify the conditions that correspond to the shaded region of the coordinate plane shown below.

A. \( x \geq 1 \) and \( y < 0 \)
B. \( x > 1 \) and \( y \leq 0 \)
C. \( x < 0 \) and \( y \geq 1 \)
D. \( x < 0 \) and \( y > 1 \)
37. If 5 times a number is increased by 4, the result is 20 less than the square of the number. Choose the equation that could be used to find this number, \( x \).

- A. \( 5x + 4 = 20 - x^2 \)
- B. \( 5(x + 4) = x^2 - 20 \)
- C. \( 5x + 4 = x^2 - 20 \)
- D. \( 9x = x^2 - 20 \)

38. Choose the equation that could be used to find the smallest of 3 consecutive integers if the sum of the smallest and twice the second is 20 more than the third. Let \( x \) be the smallest integer.

- A. \( x + 2(x + 1) = (x + 2) + 20 \)
- B. \( x + 2 + (x + 1) = (x + 2) + 20 \)
- C. \( x + 2(x + 2) = (x + 3) + 20 \)
- D. \( x + 2(x + 1) + 20 = x + 2 \)

39. For a two-digit number, let \( u = \) the units digit and \( t = \) the tens digit. Choose the equation that is equivalent to “A two-digit number is 7 times the sum of its digits.”

- A. \( tu = 7(t + u) \) \( (u = 7(t + u)) \)
- B. \( 10t + u = 7t + u \)
- C. \( 10(t + u) = 7(t + u) \)
- D. \( 10t + u = 7(t + u) \)
40. A 20-acre field produces 300 bushels of wheat. Let \( W \) represent the number of bushels produced by a 50-acre field at the same rate. Select the correct statement of the given condition.

A. \( \frac{20}{300} = \frac{50}{W} \)

B. \( \frac{20}{W} = \frac{50}{300} \)

C. \( \frac{20}{300} = \frac{W}{50} \)

D. \( \frac{20}{50} = \frac{W}{300} \)

41. It takes 3 equally skilled people 7 hours to shingle Mr. Adam's roof. Let \( t \) be the time required for only 2 of these men to do the same job. Select the correct statement of the given condition.

A. \( \frac{7}{2} = \frac{3}{t} \)

B. \( \frac{3}{2} = \frac{7}{t} \)

C. \( \frac{3}{7} = \frac{2}{t} \)

D. \( \frac{3}{2} = \frac{t}{7} \)
42. Suppose there is a 10% "bed tax" added to the cost of a hotel room. If the total bill for one night is $46.20, what is the cost of the room?

A. $46.20  
B. $46.10  
C. $42.00  
D. $36.20

43. In a sample of 30 full-time students at a particular college, 18 were also holding down a part-time job requiring at least 10 hours/week. If this proportion holds for the entire college enrollment of 12,000 students, how many full-time students at this college are actually holding down a part-time job of at least 10 hours/week?

A. 7200  
B. 4800  
C. 2000  
D. 720
ALGEBRA
PRACTICE
EXPLANATIONS

SKILLS                                      PROBLEMS
IC1  Add, subtract, multiply, and divide real numbers  1–7
IC2  Apply the order of operations agreement           8–9
IC3  Use scientific notation                          10–11
IIIC2 Use properties of operations                  12–13
IIIC2 Determine if a number is a solution to an equation or inequality  14–15
IIIC2 Use properties to identify equivalent equations and inequalities  16–19
IC4  Solve linear equations and inequalities         20–22
IC5  Use algebraic formulas                          23–25
IC6  Find values of functions                        26–27
IC7  Find factors of quadratic expressions           28–29
IC8  Find solutions to quadratic equations           30–31
IC9  Solve a system of two linear equations in two unknowns  32–34
IIIC4 Identify specified regions of the coordinate plane     35–36
IVC2  Solve problems involving the structure and logic of algebra  37–39
IIIC3 Identify statements of proportionality and variation  40–41
IVC1  Solve algebraic word problems with variables    42–43
ADD, SUBTRACT, MULTIPLY, AND DIVIDE REAL NUMBERS

1. C is the correct response. In A, coefficients were incorrectly added, followed by combining the radicands. In B, the first term was incorrectly simplified. In D, the radicals were incorrectly simplified by not getting the square root factor.

2. C is the correct response. In A, π was not accounted for at all. In B, all terms were combined even though the last term doesn’t involve π. In D, the π terms were incorrectly combined.

3. B is the correct response. In A, the coefficients were incorrectly combined without simplifying $\sqrt{9}$ first. In C, the coefficients were combined as if the $\sqrt{9}$ was just the rational number 9. In D, the 9 was squared to incorrectly eliminate the radical sign and then added.

4. A is the correct response. In B, the response was incorrectly simplified by not getting the square root of the perfect square factor. In C, the radicands were added instead of multiplied. In D, each radical was squared instead of getting the square roots.

5. A is the correct response. In B, the response was incorrectly simplified by not getting the square root of the perfect square factor. In C, the response was incorrectly simplified by obtaining a perfect square factor. In D, the term was rationalized first, but the irrational numerator was incorrectly divided with the resulting rational denominator.

6. C is the correct response. In A, the denominator was rationalized incorrectly. In B, the numerator was incorrectly simplified by failing to find the square root. In D, the terms were multiplied instead of divided.

7. C is the correct response. In A, the coefficients in the numerator were added instead of multiplied. In B, the terms were incorrectly multiplied involving π in the numerator. In D, the coefficients were incorrectly divided.
APPLY THE ORDER OF OPERATIONS AGREEMENT

8. \( B \) is the correct response. In \( A \), \( 2^3 \) was multiplied as \((2 \times 3)\) instead of \((2 \times 2 \times 2)\), thus obtaining 6 instead of 8. In \( C \), the 3 was not distributed completely. In \( D \), numbers with variable terms were incorrectly combined.

9. \( D \) is the correct response. In \( A \), the first and last terms were improperly canceled. In \( B \), addition was done first without a common denominator. In \( C \), division was correct, but the fractions were added incorrectly in the last step.

USE PROPERTIES OF OPERATIONS

10. \( C \) is the correct response. In \( A \), unlike terms were incorrectly added. In \( B \), the 8 was incorrectly distributed by not multiplying it with the second factor. In \( D \), the terms were incorrectly divided by 4.

11. \( A \) is the correct response. In \( B \), this is the correct illustration of the inverse property of addition. In \( C \), the illustration of the associative property of multiplication is correct. In \( D \), the illustration of the distributive property and the identity property of multiplication, which says that \( a(1) = a \), is correct.

USE SCIENTIFIC NOTATION

12. \( A \) is the correct response. In \( B \), the result \( 0.5 \times 10^{-10} \) was incorrectly converted to \( 0.5 \times 10^{-9} \). In \( C \), the numbers were multiplied instead of divided. In \( D \), the exponents were added instead of subtracted.

13. \( C \) is the correct response. In \( A \), a negative exponent was interpreted as a negative number. In \( B \), the decimal point was moved the wrong number of places. In \( D \), the decimal point was moved in the wrong direction.
DETERMINE IF A NUMBER IS A SOLUTION TO AN EQUATION OR INEQUALITY

14. \( B \) is the correct response. In \( A \), \( x = -1 \) is incorrectly identified as a solution for equation iii. In \( C \), \( x = -1 \) is not identified as a solution for ii. In \( D \), \( x = -1 \) is not identified as a solution for i.

15. \( B \) is the correct response. In \( A \), \( x = \frac{1}{2} \) is incorrectly identified as a solution for equation ii. In \( C \), \( x = \frac{1}{2} \) is not identified as a solution for iii. In \( D \), \( x = \frac{1}{2} \) is not identified as a solution for i.

USE PROPERTIES TO IDENTIFY EQUIVALENT EQUATIONS AND INEQUALITIES

16. \( D \) is the correct response. In \( A \), there was a sign error subtracting 5 from 1. In \( B \), the negative sign was dropped on the \( 3x \). In \( C \), the \(-5\) was “combined” with \( x \) instead of the 5 being subtracted from 1.

17. \( A \) is the correct response. In \( B \), there was a sign error subtracting 6 from both sides. In \( C \), the inequality symbol was unnecessarily reversed. In \( D \), there was a sign error subtracting \( x \) from both sides.

18. \( C \) is the correct response. In \( A \), \(-5\) was not multiplied by 4. In \( B \), only one side of the equation was multiplied by 4. In \( D \), there was a mistake in clearing the fraction on the left when both sides were multiplied by 4.

19. \( B \) is the correct response. In \( A \), the first quantity and the third inequality were not multiplied by \( y \). In \( C \), the inequality was not reversed when both sides were multiplied by a negative value. In \( D \), the equivalent inequality was transposed without changing the inequality symbols.
SOLVE LINEAR EQUATIONS AND INEQUALITIES

20. \( B \) is the correct response. In \( A \), the equivalent equation \( 11x = 14 \) was solved incorrectly. In \( C \), the equivalent equation \( 3x - 1 = 10 - 8x + 3 \) was solved by adding 1 to both the 10 and the 3 before combining. In \( D \), 2 was not distributed through the entire quantity \( (5 - 4x) \).

21. \( A \) is the correct response. In \( B \), the inequality symbol was not reversed when both sides of \(-4m < 12\) were divided by \(-4\). In \( C \), the terms \(-4m\) and 4 were divided by 4, producing \(4 - m < 0\), which is not equivalent to the original inequality. In \( D \), the constant 4 was combined with the coefficient \(-4\) on the left side.

22. \( D \) is the correct response. In \( A \), there was a sign error handling the equivalent equation \( 5x - 3 = 6x - 8 \). In \( B \), the coefficients \(-1\) and 6 were not distributed through the quantities \((3 - x)\) and \((x - 3)\), respectively. In \( C \), \(-1\) was not distributed through the entire quantity \((3 - x)\).

USE ALGEBRAIC FORMULAS

23. \( B \) is the correct response. In \( A \), there was an order of operation mistake and only the \( 77^\circ \) was multiplied by \( \frac{5}{9} \). In \( C \), the result of the subtraction was multiplied by \( \frac{9}{5} \) instead of \( \frac{5}{9} \). In \( D \), the \( C \) was substituted for rather than \( F \), and then \( F \) was solved for incorrectly.

24. \( B \) is the correct response. In \( A \), the remainder of the division was incorrect. In \( C \), there was a division error. In \( D \), the wrong variables were substituted for and multiplied.

25. \( D \) is the correct response. In \( A \), the \( r \) was subtracted rather than added to both sides. In \( B \), the coefficient \( 2x \) was subtracted rather than divided by on both sides. In \( C \), the \( 2x \) was divided before adding the \( r \) to both sides.
FIND VALUES OF FUNCTIONS

26. \(D\) is the correct response. In \(A\), the \(-3\) was not raised to the exponent. In \(B\), a \(-9\) was used for the middle term instead of \(9\). In \(C\), the \(-3\) was raised to the power of 2 incorrectly, resulting in \(6\) instead of \(9\).

27. \(B\) is the correct response. In \(A\), \(-2\) was used for the last term instead of 2. In \(C\), \((-2)^3\) was incorrectly multiplied as \(-6\) instead of \(-8\). In \(D\), \(8\) was used for the middle term instead of \(-8\).

FIND FACTORS OF QUADRATIC EXPRESSIONS

28. \(C\) is the correct response. Options \(A\), \(B\), and \(D\) are incorrect choices because they contain factors of the first and last terms, but these terms do not produce the middle term of the quadratic expression.

29. \(C\) is the correct response. Options \(A\), \(B\), and \(D\) are incorrect choices because they contain factors of the first and last terms, but these terms do not produce the middle term of the quadratic expression.

FIND SOLUTIONS TO QUADRATIC EQUATIONS

30. \(B\) is the correct response. In \(A\), numbers were correctly substituted into the quadratic formula, but there was an arithmetic mistake with the radical in one of the solutions. In \(C\), the quadratic was incorrectly factored into two linear factors. In \(D\), numbers were correctly substituted into the quadratic formula, but there was an arithmetic mistake with the radicals in both of the solutions.

31. \(C\) is the correct response. In \(A\), the quadratic was incorrectly factored into two incorrect linear factors. In \(B\), the quadratic was incorrectly factored into two incorrect linear factors. In \(D\), numbers were substituted incorrectly into the quadratic formula.
SOLVE A SYSTEM OF TWO LINEAR EQUATIONS IN TWO UNKNOWNS

32. *A* is the correct response. In *B*, there was a computational error. In *C*, there was a computational (sign) error. In *D*, both variables were eliminated.

33. *D* is the correct response. In *A*, there was an error in computation. In *B*, the solution satisfies only one of the equations. In *C*, both sides of an equation were not multiplied during the elimination process.

34. *D* is the correct response. In *A*, the statement $0 = 0$ was misinterpreted as an ordered pair $(0, 0)$. In *B*, the ordered pair $(5, 1)$ is only one of infinitely many ordered pairs in the solution set. In *C*, the statement $0 = 0$ was misinterpreted as the indication of the empty set.

IDENTIFY SPECIFIED REGIONS OF THE COORDINATE PLANE

35. *A* is the correct response. In *B*, the wrong $x$-intercept for the boundary line was found. In *C*, the wrong type of boundary line was used. It should be solid. In *D*, the wrong region was shaded.

36. *B* is the correct response. In *A*, $x \geq 1$ would require a solid vertical boundary line. In *C*, the variables for vertical and horizontal lines were reversed. In *D*, the variables for vertical and horizontal lines were reversed, and both boundaries would have to be dashed lines.

SOLVE PROBLEMS INVOLVING THE STRUCTURE AND LOGIC OF ALGEBRA

37. *C* is the correct response. In *A*, there was an incorrect order for subtraction in the phrase “20 less than the square of the number.” In *B*, there was an order of operations error in the phrase “5 times a number is increased by 4.” In *D*, there was a misinterpretation of the phrase “increased by 4.”

38. *A* is the correct response. In *B*, there was an incorrect interpretation of the phrase “twice the second.” In *C*, there were incorrect representations for the second and third integers. In *D*, there was an incorrect placement of the equal sign.
39. \( D \) is the correct response. In \( A \), there was an incorrect representation of the two-digit number; \( tu \) would mean \( t \) times \( u \). In \( B \), there was an incorrect representation of the phrase “7 times the sum of its digits.” In \( C \), there was an incorrect representation of the two-digit number.

40. \( A \) is the correct response. In \( B \), 300 bushels is incorrectly associated with the 20-acre field. In \( C \), a proportion is incorrectly set up for inverse variation. In \( D \), equivalent to \( B \), there is an inverse variation setup.

41. \( D \) is the correct response. In \( A \), 7 hours is incorrectly associated with 2 men. In \( B \), a proportion for direct variation is incorrectly set up. In \( C \), equivalent to \( B \), there is a direct variation setup.

42. \( C \) is the correct response. In \( A \), the additional bed tax was ignored. In \( B \), the equation was incorrectly set up as \( x + 0.10 = \$46.20 \). In \( D \), the equation was incorrectly set up as \( x + 10 = \$46.20 \).

43. \( A \) is the correct response. In \( B \), the proportion was incorrectly set up as \( \frac{12}{30} = \frac{x}{12,000} \).

In \( C \), the proportion was incorrectly set up as \( \frac{18}{30} = \frac{12,000}{x} \). This produced \( x = 20,000 \). This answer was divided by the irrelevant value 10. In \( D \), the value 1200 was used in the proportion instead of 12,000.