CHAPTER 5
LOGIC

SKILLS  Overview  326
        IE1  Deduce facts of set inclusion and noninclusion from diagrams  327
        IIE1  Identify negations of simple and compound statements  330
        IIE2  Determine equivalence or nonequivalence of statements  335
        IIE2  Identify rules for transforming statements  338
        IIE3  Draw logical conclusions from data  340
        IIE4  Identify invalid arguments that have true conclusions  347
        IIE1  Identify valid reasoning patterns  351
        IVE1  Draw logical conclusions when facts warrant them  354

PRACTICE PROBLEMS  361

PRACTICE EXPLANATIONS  371
Logic Skills

Logic is a science that deals with the formal principles of reasoning. The ability to reason abstractly is important for many situations encountered in daily life. Understanding logic helps us recognize faulty arguments, analyze problems, and gain confidence that the decisions we make are based on valid conclusions.

The titles of the units found in this chapter are listed below in italics. Under each unit title is listed the specific logic skill or skills covered in that unit.

Deduce facts of set inclusion and noninclusion from diagrams.
Skill IE1: The student will deduce facts of set-inclusion or non-set-inclusion from a diagram.

Identify negations of simple and compound statements.
Skill IIIE1: The student will identify statements equivalent to the negations of simple and compound statements.

Determine equivalence or nonequivalence of statements.
Skill IIIE2: The student will determine equivalence or non-equivalence of statements.

Identify rules for transforming statements.
Skill IIIE2: The student selects applicable rules for transforming statements without affecting their meaning.

Draw logical conclusions from data.
Skill IIIE3: The student will draw logical conclusions from data.

Identify invalid arguments that have true conclusions.
Skill IIIE4: The student will recognize that an argument may not be valid even though its conclusion is true.

Identify valid reasoning patterns.
Skill IIIE1: The student recognizes valid reasoning patterns as illustrated by valid arguments in everyday language.

Draw logical conclusions when facts warrant them.
Skill IVE1: The student draws logical conclusions when the facts warrant them.
DEDUCE FACTS OF SET INCLUSION AND NONINCLUSION FROM DIAGRAMS

The problems in this section involve deducing facts of set inclusion or set noninclusion from a diagram.

A set is a collection of objects. Sets are named using capital letters. The chart below illustrates three relationships that may exist between the members of two given sets.

Reminder: Circle A represents set A. The size of each circle is not proportional to the size of the set it represents.

<table>
<thead>
<tr>
<th>Relationship</th>
<th>*Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each member of set A is also a member of set B. Set B completely overlaps set A.</td>
<td><img src="U_A_B" alt="Diagram" /></td>
</tr>
<tr>
<td>At least one member of set A is also a member of set B. Set A partially overlaps set B.</td>
<td><img src="U_A_B" alt="Diagram" /></td>
</tr>
<tr>
<td>No member is a member of set A and set B. Set A does not overlap set B.</td>
<td><img src="U_A_B" alt="Diagram" /></td>
</tr>
</tbody>
</table>

*Assume that no regions are empty.

The U in the diagram is a symbol for the universal set, which contains all the members for a particular discussion.

Examples

1. Set A is the set of chemistry majors at Happy College.
   Set B is the set of all science majors at Happy College.
   Set U is the set of all students at Happy College.

   A diagram showing this relationship between A and B follows.
Skill IE1  *Deduce facts of set inclusion and noninclusion from diagrams*

What can you assume about the relationship between set A and set B?

Because set B completely overlaps set A, each member of set A is also a member of set B.

2. Set A is the set of students at Happy College who play baseball.  
Set B is the set of students at Happy College who take English.  
Set U is the set of all students at Happy College.  

It is known that some baseball players at Happy College take English.  
A diagram showing the relationship between set A and set B follows.  
Note that no regions are empty.

What can you assume about the relationship between set A and set B?

Set A and set B are overlapping sets. This means that at least one member of set A is also a member of set B.

3. Set A is the set of students at Happy College who play on the soccer team.  
Set B is the set of students at Happy College who play on the baseball team.  
Set U is the set of all students at Happy College.  

A diagram showing the relationship between A and B follows.

What can you assume about the relationship between set A and set B?

Set A and set B do not overlap. No member of set A is also a member of set B.
Skill IE1  

Deduce facts of set inclusion and noninclusion from diagrams

4. Sets A, B, C, and U are related as shown in the diagram. Assume that not one of the five regions is empty.

Which of the following statements are true about this particular diagram?

A. Any element that is a member of set A is also a member of set B.
B. No element is a member of all three sets: A, B, and C.
C. Any element that is a member of set A is also a member of set U.
D. No element is a member of set A and set C.

All of the statements are true.

Reminder: On the CLAST and most multiple-choice examinations, only one option will be true.

5. Sets A, B, C, and U are related as shown in the diagram.

Which of the following statements is true? Assume that not one of the six regions is empty.

A. Any element that is a member of set A is also a member of set B.
B. No element is a member of all three sets: A, B, and C.
C. Any element that is a member of set U is also a member of set C.
D. None of the above statements is true.

B is the correct response. In response A, set A only partially overlaps set B, so there are elements in set A that are not in set B. In response C, there are elements outside of set C that are part of set U. Response D is incorrect because response B is true.
IDENTIFY NEGATIONS OF SIMPLE AND COMPOUND STATEMENTS

In the study of logic, a statement is a sentence that can be assigned a truth value of either true or false. The sentence “It is raining outside” is a statement because you can look outside and determine whether it is raining or not raining. Therefore, a truth value of true or false can be assigned to the statement. In the study of logic, the sentence “Please be quiet” is not a statement. It is a request or a command. A truth value of true or false cannot be assigned to the sentence.

Statements may be classified as either simple or compound. A simple statement is a single statement with no connective. A compound statement is two or more statements joined by connectives such as and, or, or if, then. “It is raining outside” is a simple statement, while “It is raining outside and the sun is shining” is a compound statement.

Even though putting statements in symbolic form is not necessary, often it is extremely helpful to use letters to represent the statements that are given in words. In algebra, the letters x, y, and z are used to represent variables, and the letters a, b, and c are used to represent constants. In logic, the letters p, q, and r usually represent statements.

In this section, the identification of statements that are equivalent to the negations of simple and compound statements will be discussed. To negate a statement means to give the opposite truth value. That is, if the statement is true, then its negation is false. If the statement is false, then its negation is true.

In finding the negations of simple and compound statements, use the following rules:

**Rule 1.**

- **Statement**: It is raining.
- **Negation**: It is not raining.

**Rule 2.**

- **Statement**: It is not raining.
- **Negation**: It is raining.

In the negation of a not statement, the not is dropped.

**Rule 3.**

- **Connective Statement**: And
- **Negation**: It is raining outside and the sun is shining.

- **Connective Statement**: or
- **Negation**: It is not raining outside or the sun is not shining.

The negation of an and statement becomes an or statement, and each of the statements that make up the compound statement is also negated.
Skill HE1  Identify negations of simple and compound statements

**Rule 4.**

connective statement  Or
negation  Pat plays baseball or he plays football.

Pat does not play baseball and he does not play football.

The negation of an or statement becomes an and statement, and each of the statements that make up the compound statement is also negated.

**Rule 5.**

connective statement  If, then
negation  If it snows, then I will be cold.

It snows and I am not cold.

The negation of an if, then statement is no longer an if, then statement. It is the first statement followed by the connective and then the negation of the second statement.

A summary of the negations of simple and compound statements is given in the following table:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. not p.</td>
<td>2. p.</td>
</tr>
<tr>
<td>3. p and q.</td>
<td>3. (not p) or (not q).</td>
</tr>
<tr>
<td>4. p or q.</td>
<td>4. (not p) and (not q).</td>
</tr>
<tr>
<td>5. If p, then q.</td>
<td>5. (p) and (not q).</td>
</tr>
</tbody>
</table>
**Skill IIIE1**  *Identify negations of simple and compound statements*

**Examples**

The table below contains some examples of statements and their negations.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. It is cold <em>and</em> I will turn the heater on.</td>
<td>1. It is not cold <em>or</em> I will not turn the heater on.</td>
</tr>
<tr>
<td>2. <em>If</em> it rains, <em>then</em> the boys will go to the mall.</td>
<td>2. It rains <em>and</em> the boys will not go to the mall.</td>
</tr>
<tr>
<td>3. I will write a paper <em>or</em> I will not pass this class.</td>
<td>3. I will not write a paper <em>and</em> I will pass this class.</td>
</tr>
<tr>
<td>4. It is <em>not</em> snowing.</td>
<td>4. It is snowing.</td>
</tr>
<tr>
<td>5. <em>If</em> I play table tennis, <em>then</em> I always win.</td>
<td>5. I play table tennis <em>and</em> I do not always win.</td>
</tr>
<tr>
<td>6. I am going to the mountains <em>and</em> I will see snow.</td>
<td>6. I am not going to the mountains <em>or</em> I will not see snow.</td>
</tr>
<tr>
<td>7. Joe has a headache <em>or</em> he is playing basketball.</td>
<td>7. Joe does not have a headache <em>and</em> he is not playing basketball.</td>
</tr>
</tbody>
</table>

Some statements used in logic are quantified. A **quantifier** is a word used to tell how many. Quantified statements use the words *all, some, some—not, and no*. A statement is said to be quantified when these terms are used along with the statement. The following are some examples of quantified statements: (1) All dogs have fleas. (2) Some boys do not play baseball. (3) No girl plays football. (4) Some teachers are mathematics teachers.

The following diagrams illustrate the meanings of quantified statements.

**Negation**

*All A's are B's.*  
![Diagram](image1)

*Some A's are not B's.*  
![Diagram](image2)

*Some A's are B's.*  
![Diagram](image3)

*No A's are B's.*  
![Diagram](image4)
Skill IIIE1  Identify negations of simple and compound statements

In finding the negations of statements with quantifiers, use the following rules:

**Rule 1.**  
quantifier statement  
All  
All chefs can cook.

negation  
Some chefs can *not* cook.

The negation of a statement containing the quantifier *all* is *some*—*not*.

**Rule 2.**  
quantifier statement  
Some  
Some children play soccer.

negation  
No children play soccer.

The negation of a statement containing the quantifier *some* is *no*.

**Rule 3.**  
quantifier statement  
Some—*not*  
Some people do *not* like to go fishing.

negation  
All people like to go fishing.

The negation of a statement containing the quantifier *some—* *not* *is all*.

**Rule 4.**  
quantifier statement  
No  
No animal has one leg.

negation  
Some animals have one leg.

The negation of a statement containing the quantifier *no* is *some*. A summary of the negations of quantified statements is given in the table on the next page.
Skill HE1  Identify negations of simple and compound statements

The following diagram and table may be useful for negating statements that contain quantifiers.

Some are  
\[ \begin{array}{c}
\text{Negation} \\
\text{Negation}
\end{array} \]

All  
Some are not  
No

<table>
<thead>
<tr>
<th>Statement</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All are p.</td>
<td>1. Some are not p.</td>
</tr>
<tr>
<td>2. Some are p.</td>
<td>2. None are p.</td>
</tr>
<tr>
<td>3. Some are not p.</td>
<td>3. All are p.</td>
</tr>
<tr>
<td>4. None are p.</td>
<td>4. Some are p.</td>
</tr>
</tbody>
</table>

Examples

The table below contains some quantified statements and their negations.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Some fish do not have skin.</td>
<td>1. All fish have skin.</td>
</tr>
<tr>
<td>2. No roses are red.</td>
<td>2. Some roses are red.</td>
</tr>
<tr>
<td>3. All pansies are multicolored.</td>
<td>3. Some pansies are not multicolored.</td>
</tr>
<tr>
<td>4. Some students do not take history.</td>
<td>4. All students take history.</td>
</tr>
<tr>
<td>5. Some cats have blue eyes.</td>
<td>5. No cat has blue eyes.</td>
</tr>
<tr>
<td>6. No trees have green leaves.</td>
<td>6. Some trees have green leaves.</td>
</tr>
</tbody>
</table>
**SKILL H2E2  Determine equivalence or nonequivalence of statements**

**DETERMINE EQUIVALENCE OR NONEQUIVALENCE OF STATEMENTS**

Two statements may or may not be equivalent. Equivalent statements are those that have the same logical meaning. The following table shows statements that are equivalent in meaning:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Equivalent Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>p and q.</td>
<td>q and p.</td>
</tr>
<tr>
<td>p or q.</td>
<td>q or p.</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>If not q, then not p.</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>(not p) or (q).</td>
</tr>
<tr>
<td>not (p and q).</td>
<td>(not p) or (not q).</td>
</tr>
<tr>
<td>not (p or q).</td>
<td>(not p) and (not q).</td>
</tr>
<tr>
<td>not (if p, then q).</td>
<td>(p) and (not q).</td>
</tr>
<tr>
<td>not (some are p).</td>
<td>None are p.</td>
</tr>
<tr>
<td>not (all are p).</td>
<td>Some are not p.</td>
</tr>
<tr>
<td>not (some are not p).</td>
<td>All are p.</td>
</tr>
<tr>
<td>not (none are p).</td>
<td>Some are p.</td>
</tr>
<tr>
<td>not [(some are p) and (all are q)].</td>
<td>(none are p) or (some are not q).</td>
</tr>
<tr>
<td>not [(none are p) or (some are q)].</td>
<td>(some are p) and (none are q).</td>
</tr>
</tbody>
</table>

The **converse** of “if p, then q” is “if q, then p.”

The **inverse** of “if p, then q” is “if not p, then not q.”

The **contrapositive** of “if p, then q” is “if not q, then not p.”

The converse and inverse of “if p, then q” are *not* logically equivalent to “if p, then q.”
Skill IIE2  *Determine equivalence or nonequivalence of statements*

**Rule 1:** If asked to find a statement that is NOT logically equivalent to “if p, then q,” look for one of these forms:

1. converse “if q, then p”
2. inverse “if not p, then not q”

**Rule 2:** If asked to find a statement that is logically equivalent to “if p, then q,” look for one of these forms:

1. contrapositive “if not q, then not p”
2. “(not p) or (q)”

**Examples**

1. Write two statements that are logically equivalent to “If Jo passes algebra, then she will enroll in trigonometry.”

   **Form 1:** contrapositive “if not q, then not p”
   If Jo does not enroll in trigonometry, then she did not pass algebra.

   **Form 2:** “(not p) or (q)”
   Jo does not pass algebra or she enrolls in trigonometry.

2. Write two statements that are NOT logically equivalent to “If Terry eats ice cream, then he will gain weight.”

   **Form 1:** “if not p, then not q” (inverse)
   If Terry does not eat ice cream, then he will not gain weight.

   **Form 2:** “if q, then p” (converse)
   If Terry gains weight, then he eats ice cream.

3. Write a statement that is logically equivalent to “It is not true that Tom is not a lawyer or Alice is a doctor.”

   The form of this statement is “not (p or q).”
   Therefore, the equivalent form is “(not p) and (not q).”

   Tom is a lawyer and Alice is not a doctor.

   **Reminder:** When the phrase “It is not true” precedes a compound statement, it means to negate the compound statement.
Skill IIIE2  Determine equivalence or nonequivalence of statements

4. Select the statement below that is NOT logically equivalent to “If Joe accepts the position, then he will purchase a new car.”

   A. If Joe purchases a new car, then he accepted the position.
   B. Joe purchased a new car or he did not accept the position.
   C. If Joe did not purchase a new car, then he did not accept the position.
   D. Joe did not accept the position or he purchased a new car.

   The answer is A, which is the converse of the given statement.

5. Select the statement below that is logically equivalent to “It is not true that Mike may go to the mall if he cleans his room.”

   A. Mike cleans his room or he does not go to the mall.
   B. Mike does not clean his room and he does not go to the mall.
   C. Mike cleans his room and he does not go to the mall.
   D. Mike does not clean his room or he does not go to the mall.

   The answer is C since “not (q, if p)” is “(p) and (not q).”

6. Write a statement that is logically equivalent to “It is not true that all students play tennis and some students have calculators.”

   Form of statement: not [(all are p) and (some are q)].
   Equivalent form: (some are not p) or (none are q).
   Some students are not playing tennis or no students have calculators.
IDENTIFY RULES FOR TRANSFORMING STATEMENTS

Statements may be transformed (rewritten in a different form) without affecting their meaning. The rules for transformation are illustrated in the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Equivalent Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>not (not p).</td>
<td>p.</td>
</tr>
<tr>
<td>not (p and q).</td>
<td>(not p) or (not q).</td>
</tr>
<tr>
<td>not (p or q).</td>
<td>(not p) and (not q).</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>(not p) or (q).</td>
</tr>
<tr>
<td>Not all are p.</td>
<td>Some are not p.</td>
</tr>
<tr>
<td>All are not p.</td>
<td>None are p.</td>
</tr>
<tr>
<td>Not (if p, then q).</td>
<td>(p) and (not q).</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>If not q, then not p.</td>
</tr>
</tbody>
</table>

Examples

1. Select the rule from the table above that transforms statement “i” into statement “ii.”
   i. If today is Saturday, then Joe will play golf.
   ii. Today is not Saturday or Joe plays golf.

   Form for i: If p, then q.
   Form for ii: (not p) or (q).

   The rule from the table that transforms statement “i” into statement “ii” is “if p, then q’ is equivalent to ‘(not p) or (q).’”

2. Select the rule from the table above that transforms statement “i” into statement “ii.”
   i. Not all people cook.
   ii. Some people do not cook.

   Form for i: not all are p.
   Form for ii: Some are not p.

   The rule from the table that transforms statement “i” into statement “ii” is “not all p’ is equivalent to ‘some are not p.’”
Skill IIIIE2: Identify rules for transforming statements

3. Select the rule from the table on the previous page that transforms statement "i" into statement "ii."
   i. It is not true that it is raining and Susan is playing golf.
   ii. It is not raining or Susan is not playing golf.

   Form for i: \( \neg (p \land q) \).
   Form for ii: \( \neg p \lor \neg q \).

   The rule from the table that transforms statement "i" into statement "ii" is "\( \neg (p \land q) \)’ is equivalent to \( \neg p \lor \neg q \).”

4. Write the statements that illustrate the rule “‘If p, then q’ is equivalent to ‘If not q, then not p,’” using \( p \) to represent “The temperature is 30 degrees” and \( q \) to represent “Hyo will wear a coat.”

   “If the temperature is 30 degrees, then Hyo will wear a coat” is equivalent to “If Hyo does not wear a coat, then the temperature is not 30 degrees.”

5. Select the rule of logical equivalence that directly (in one step) transforms statement “i” into statement “ii."
   i. It is not true that Tom is sleeping and the alarm is ringing.
   ii. Tom is not sleeping or the alarm is not ringing.

   A. “\( \neg (p \lor q) \)” is equivalent to “\( \neg p \lor \neg q \).”
   B. “\( \neg (p \lor q) \)” is equivalent to “\( p \).”
   C. “\( \neg (p \land q) \)” is equivalent to “\( \neg p \lor \neg q \).”
   D. “\( \neg \text{all are } p \)” is equivalent to “\( \text{some are not } p \).”

   \( C \) is the correct response. The form for statement “i” is “\( \neg (p \land q) \)” and the form for statement “ii” is “\( \neg p \lor \neg q \).”
DRAW LOGICAL CONCLUSIONS FROM STATEMENTS

Many situations require that we draw conclusions about given information. To analyze such data, it is helpful to consider the information and determine what fits in the category described. The examples below illustrate this process.

Examples

1. Read the requirements and each applicant’s qualifications for securing a managerial position with a major oil company. Then identify which of the applicants could qualify for the position.

To qualify for the position an applicant must have been with the company at least 10 years and have a bachelor’s degree in business administration.

Mr. Blue has been with the company 11 years. He has a bachelor’s degree in history.

Mrs. May has 8 years with the company and has a bachelor’s degree in business administration.

Joe Green has 12 years with the company and has a bachelor’s degree in business administration.

Joe Green is the applicant who qualifies. Mrs. May does not qualify since she meets only one of the requirements. When a statement contains the connective and, both of the qualifications must be met. Mr. Blue also does not qualify for the job because he satisfies only one of the requirements.

2. Read the requirements and each applicant’s qualifications for obtaining a $120,000 loan needed to buy a house. Then identify which of the applicants would qualify for the loan.

To qualify for a home loan of $120,000 an applicant must have a gross income of $40,000 if single ($60,000 combined income if married) and debts of no more than $5,000.

Mr. and Mrs. Smith are married with 3 children. She makes $32,000, and her husband makes $33,000. Their only debts are $2,000 to their bank charge card and a $5,000 balance on their car loan.

Mr. and Mrs. Jones are married and have no children. Mr. Jones makes $65,000 and his wife does not work outside the home. Their only debt is the $3,000 owed on their car.

Bob Adams is single and makes $35,000. He is free of debt.
Mr. Jones is the person who qualifies. Mrs. Smith and her husband have the required income but their debts are too high. Bob Adams does not meet the income requirement even though he has no debts.

Such information is presented in statements using quantifiers, conjunctions (and), disjunctions (or), implications (if, then), and negations of these forms. Two premises will be given that are assumed to be true. From the given premises, sometimes a logical conclusion can be determined.

Conclusions

In order to draw a logical conclusion, the use of one or more of the following logical argument forms may be desirable. The table on page 342 gives the argument both in words and in form. In an if, then statement, the statement following if is called the hypothesis, the statement following then is called the conclusion.
<table>
<thead>
<tr>
<th>Argument</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given that</strong></td>
<td><strong>Form</strong></td>
</tr>
<tr>
<td>It rains or the sun shines.</td>
<td>Given that</td>
</tr>
<tr>
<td>It does not rain.</td>
<td>p or q.</td>
</tr>
<tr>
<td>The sun shines.</td>
<td>not p.</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td>q.</td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>It rains or the sun shines.</td>
<td>p or q.</td>
</tr>
<tr>
<td>The sun does not shine.</td>
<td>not q.</td>
</tr>
<tr>
<td>It rains.</td>
<td>p.</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td>q.</td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>If it rains, then the sun shines.</td>
<td>If p, then q.</td>
</tr>
<tr>
<td>It rains.</td>
<td>p.</td>
</tr>
<tr>
<td>The sun shines.</td>
<td>q.</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td></td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>If it rains, then the sun shines.</td>
<td>If p, then q.</td>
</tr>
<tr>
<td>The sun does not shine.</td>
<td>not q.</td>
</tr>
<tr>
<td>It does not rain.</td>
<td>not p.</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td></td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>If it rains, then the sun shines.</td>
<td>If p, then q.</td>
</tr>
<tr>
<td>If the sun shines, then I go to the beach.</td>
<td>If q, then r.</td>
</tr>
<tr>
<td>If it rains, then I go to the beach.</td>
<td></td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td></td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>No fractions are integers.</td>
<td>If p, then not q.</td>
</tr>
<tr>
<td>(If numbers are fractions, then they are not integers.)</td>
<td></td>
</tr>
<tr>
<td>All decimal numbers are fractions.</td>
<td></td>
</tr>
<tr>
<td>(If numbers are decimals, then they are fractions.)</td>
<td></td>
</tr>
<tr>
<td>No decimal numbers are integers.</td>
<td>If r, then p.</td>
</tr>
<tr>
<td>(If numbers are decimals, then they are not integers.)</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>If r, then not q.</td>
</tr>
<tr>
<td>Given that</td>
<td></td>
</tr>
<tr>
<td>All baseball players are busy.</td>
<td>If p, then q.</td>
</tr>
<tr>
<td>(If a person plays baseball, then she or he is busy.)</td>
<td></td>
</tr>
<tr>
<td>All busy people are tired.</td>
<td>If q, then r.</td>
</tr>
<tr>
<td>(If a person is busy, then she or he is tired.)</td>
<td></td>
</tr>
<tr>
<td>All baseball players are tired.</td>
<td>If p, then r.</td>
</tr>
</tbody>
</table>
In drawing conclusions about statements, use the following method:

**Step 1.** Choose a letter to represent each statement.

**Step 2.** Determine the form the statements follow.

**Step 3.** Write the conclusion in word form.

**Examples**

3. Given that
   
i. Joe goes to a movie or he goes to the beach.
   ii. Joe does not go to the beach.

determine which conclusion can be logically deduced.

A. Joe does not go to a movie.
B. Joe goes to a movie.
C. Joe goes to the beach.
D. None of the above is true.

**Step 1.** Choose a letter to represent each statement.

   i. Joe goes to a movie or he goes to the beach.
      
      \[ p \text{ (Joe goes to a movie)} \lor q \text{ (Joe goes to the beach)} \]

   ii. Joe does not go to the beach.
      
      \[ \neg q \text{ (Joe goes to the beach)} \]

**Step 2.** Determine the form the statements follow.

These statements fit the form:

\[
\begin{array}{c}
p \lor q \\ 
\neg q \\ 
p
\end{array}
\]

**Step 3.** Write the conclusion in word form.

Joe goes to a movie.

Therefore, the conclusion is **B**.
4. Given that
   i. If Maria likes math, then she will become an engineer.
   ii. Maria likes math.

determine the conclusion that can be logically deduced.

*Step 1.* Choose a letter to represent each statement.

   i. If Maria likes math, then she will become an engineer.
      If p (Maria likes math), then q (Maria will become an engineer).

   ii. Maria likes math.
      p (Maria likes math).

*Step 2.* Determine the form the statements follow.

These statements fit the form: If p, then q.

\[
\begin{array}{cc}
\text{p} & \text{q} \\
\hline
\end{array}
\]

*Step 3.* Write the conclusion in word form.

The conclusion is “Maria will become an engineer.”

5. Given that
   i. If Emmett sings, then he will not play the piano.
   ii. Emmett plays the piano.

determine the conclusion that can be logically deduced.

*Step 1.* Choose a letter to represent each statement.

   i. If Emmett sings, then he will not play the piano.
      If p (Emmett sings), then q (Emmett will not play the piano).

   ii. Emmett plays the piano.
      not q (Emmett will not play the piano).

*Step 2.* Determine the form the statements follow.

These statements fit the form: If p, then q.

\[
\begin{array}{cc}
\text{not q} & \text{not p} \\
\hline
\end{array}
\]
Skill IIE3

Draw logical conclusions from data

Step 3. Write the conclusion in word form.

The conclusion is “Emmett does not sing.”

6. Given that
   i. If it rains, I will go to the movie.
   ii. If I go to the movie, then I don’t clean my room.

determine the conclusion that can be logically deduced.

Step 1. Choose a letter to represent each statement.

   i. If it rains, I will go to the movie.
      If p (It rains), then q (I will go to the movie).

   ii. If I go to the movie, then I don’t clean my room.
      If q (I will go to the movie), then r (I don’t clean my room).

Step 2. Determine the form the statements follow.

These statements fit the form: If p, then q.
If q, then r.
If p, then r.

Step 3. Write the conclusion in word form.

The conclusion is “If it rains, then I don’t clean my room.”

7. Given that
   i. No people are happy.
   ii. All girls are people.

determine the conclusion that can be logically deduced.

Step 1. Choose a letter to represent each statement.

   i. No people are happy.
      If you are a person (p), then you are not happy (q).

   ii. All girls are people.
      If you are a girl (r), then you are a person (p).
Skill HIE3

**Draw logical conclusions from data**

**Step 2.** Determine the form the statements follow.

These statements fit the form:
- If $p$, then $q$.
- If $r$, then $p$.

Conclusion: If $r$, then $q$.

**Step 3.** Write the conclusion in word form.

The conclusion is “If you are a girl, then you are not happy” or “No girls are happy.”

8. **Given that**

   i. All teachers are funny people.
   ii. All funny people are comedians.

   determine the conclusion that can be logically deduced.

**Step 1.** Choose a letter to represent each statement.

   i. All teachers are funny people.
      
      If you are a teacher ($p$), then you are funny ($q$).

   ii. All funny people are comedians.
      
      If you are funny ($q$), then you are a comedian ($r$).

**Step 2.** Determine the form the statements follow.

These statements fit the form:
- If $p$, then $q$.
- If $q$, then $r$.

Conclusion: If $p$, then $r$.

**Step 3.** Write the conclusion in word form.

The conclusion is “If you are a teacher, then you are a comedian” or “All teachers are comedians.”
IDENTIFY INVALID ARGUMENTS THAT HAVE TRUE CONCLUSIONS

An argument is made up of premises and the conclusion. A premise is an assumption or observation. An argument is said to be valid if its conclusion is a necessary logical consequence of its premises; otherwise, the argument is said to be invalid. The chain of reasoning that follows in the examples below consists of two premises. When the premises are accepted and you are forced to accept the conclusion, the argument is said to be valid. Note that the validity of the argument does not depend on the question of whether the conclusion is true or false. The conclusion may be true even though the chain of reasoning is incorrect.

The following chart shows two examples of arguments that are valid and a diagram illustrating each argument:

<table>
<thead>
<tr>
<th>Form of Argument</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>All A’s are B’s.</td>
<td>![Diagram A and B]</td>
</tr>
<tr>
<td>All A’s are B’s. All B’s are C’s.</td>
<td>![Diagram C and A and B]</td>
</tr>
<tr>
<td>Therefore, all A’s are C’s.</td>
<td></td>
</tr>
</tbody>
</table>

Both arguments listed in the table force you to accept the conclusion when the premises are accepted. The challenge in showing an argument is invalid is to find a way to construct a diagram in which both premises hold but the conclusion does not necessarily follow.

Reminder: Remember that even though a conclusion is a true statement, it does not mean that the argument is valid.

Examples

1. Determine whether the following argument is valid.

   All rational numbers are real numbers. Seventeen is a rational number. Therefore, 17 is a real number.
Skill IIE4  Identify invalid arguments that have true conclusions

A diagram follows to determine if the argument is valid.

Premise 1:  All rational numbers are real numbers.

Premise 2:  Seventeen is a rational number.

Conclusion: You are forced to accept the conclusion that 17 is a real number. Therefore, the argument is valid.

2. Determine whether the following argument is valid.

All rational numbers are real numbers. Seventeen is a real number. Therefore, 17 is a rational number.

A diagram follows to determine whether the argument is valid.

Premise 1:  All rational numbers are real numbers.
Skill IIE4 Identify invalid arguments that have true conclusions

Premise 2: Seventeen is a real number.

Conclusion: Because there are two diagrams that support the premises, you are not forced to accept the conclusion that 17 is a rational number. Therefore, the argument is not valid.

3. Determine whether the following argument is valid.

All integers are rational numbers. All rational numbers are real numbers. Therefore, all integers are real numbers.

A diagram follows to determine whether the argument is valid or invalid.

Premise 1: All integers are rational numbers.

Premise 2: All rational numbers are real numbers.

Conclusion: You are forced to accept the conclusion that all integers are real numbers. Therefore, the argument is valid.
4. Determine whether the following argument is valid.

All integers are real numbers. All rational numbers are real numbers. Therefore, all integers are rational numbers.

A diagram follows to determine if the argument is valid.

Premise 1: All integers are real numbers.

Premise 2: All rational numbers are real numbers.

Conclusion: Because there are several diagrams that support the premises, you are not forced to accept the conclusion that all integers are rational numbers. Therefore, the argument is invalid.
IDENTIFY VALID REASONING PATTERNS

Remember that an argument is made up of premises and a conclusion. A premise is an assumption, law, rule, widely held idea, or observation. In drawing conclusions from given premises, the premises are assumed to be true. An argument is valid if the conclusion follows from the premises. An argument is invalid if the conclusion does not follow from a valid reasoning pattern.

The following reasoning patterns are to be used when determining which logical conclusion will make the argument valid:

**Pattern I:**

<table>
<thead>
<tr>
<th>Premise 1:</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2:</td>
<td>p.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>q.</td>
</tr>
</tbody>
</table>

**Pattern II:**

<table>
<thead>
<tr>
<th>Premise 1:</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2:</td>
<td>not q.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>not p.</td>
</tr>
</tbody>
</table>

**Pattern III:**

<table>
<thead>
<tr>
<th>Premise 1:</th>
<th>p or q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2:</td>
<td>not p.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>q.</td>
</tr>
</tbody>
</table>

**Pattern IV:**

<table>
<thead>
<tr>
<th>Premise 1:</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2:</td>
<td>If q, then r.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>If p, then r.</td>
</tr>
</tbody>
</table>

**Examples**

1. Given the following premises, determine the pattern and express the conclusion.

   If I am sick, then I must take medicine. I am sick.

   p: I am sick.
   q: I must take medicine.

   Pattern: 
   
<table>
<thead>
<tr>
<th>Premise 1:</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2:</td>
<td>p.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>q.</td>
</tr>
</tbody>
</table>

   This is an example of Pattern I.

   Conclusion expressed in words: I must take medicine.
2. Given the following premises, determine the pattern and express the conclusion.
If I work hard, then I will get a promotion. I did not get a promotion.

\( p: \) I work hard.
\( q: \) I will get a promotion.

\[
\begin{array}{c|c|c}
\text{Pattern:} & \text{premise 1} & \text{If } p, \text{ then } q. \\
& \text{premise 2} & \text{not } q. \\
& \text{conclusion} & \text{not } p. \\
\end{array}
\]

This is an example of Pattern II.

Conclusion expressed in words: I did not work hard.

3. Given the following premises, determine the pattern and express the conclusion.
I play the piano or the violin. I don’t play the piano.

\( p: \) I play the piano.
\( q: \) I play the violin.

\[
\begin{array}{c|c|c}
\text{Pattern:} & \text{premise 1} & p \text{ or } q. \\
& \text{premise 2} & \text{not } p. \\
& \text{conclusion} & q. \\
\end{array}
\]

This is an example of Pattern III.

Conclusion expressed in words: I play the violin.

4. Given the following premises, determine the pattern and express the conclusion.
If I attend school, then I will graduate. If I graduate, then I will get a job.

\( p: \) I attend school.
\( q: \) I will graduate.
\( r: \) I will get a job.

\[
\begin{array}{c|c|c}
\text{Pattern:} & \text{premise 1} & \text{If } p, \text{ then } q. \\
& \text{premise 2} & \text{If } q, \text{ then } r. \\
& \text{conclusion} & \text{If } p, \text{ then } r. \\
\end{array}
\]

This is an example of Pattern IV.

Conclusion expressed in words: If I attend school, then I will get a job.
5. Given the following premises, determine the pattern and express the conclusion.

If all students study, then no failing grades are given. Some failing grades are given.

\[ p: \text{All students study.} \]
\[ q: \text{No failing grades are given.} \]
\[ \text{not } p: \text{Some students do not study.} \text{ (Remember: The negation of “all” is “some—not.”)} \]
\[ \text{not } q: \text{Some failing grades are given.} \text{ (Remember: The negation of “no” is “some.”)} \]

Pattern: \[
\begin{array}{c}
\text{premise 1} \\
\text{premise 2} \\
\text{conclusion}
\end{array}
\]

\[
\begin{array}{c}
\text{If } p, \text{ then } q. \\
\text{not } q. \\
\text{not } p.
\end{array}
\]

This is an example of Pattern II.

Conclusion expressed in words: Some students do not study.

6. Given the following premises, determine the pattern and express the conclusion.

If no person passes the test, then some questions were unfair. If some questions were unfair, then the teacher should be fired.

\[ p: \text{No person passes the test.} \]
\[ q: \text{Some questions were unfair.} \]
\[ r: \text{The teacher should be fired.} \]

Pattern: \[
\begin{array}{c}
\text{premise 1} \\
\text{premise 2} \\
\text{conclusion}
\end{array}
\]

\[
\begin{array}{c}
\text{If } p, \text{ then } q. \\
\text{If } q, \text{ then } r. \\
\text{If } p, \text{ then } r.
\end{array}
\]

This is an example of Pattern IV.

Conclusion expressed in words: If no person passes the test, then the teacher should be fired.
Some logic problems consist of an argument with two to four premises. The premises may involve simple statements, compound statements, or statements that contain negations or quantifiers. The conclusion must logically follow these premises.

The following table summarizes statements and the equivalent forms of the statements:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Equivalent Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>not (p and q).</td>
<td>(not p) or (not q).</td>
</tr>
<tr>
<td>not (p or q).</td>
<td>(not p) and (not q).</td>
</tr>
<tr>
<td>not (If p, then q).</td>
<td>p and (not q).</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>(not p) or q.</td>
</tr>
<tr>
<td>If p, then q.</td>
<td>(not q) implies (not p).</td>
</tr>
<tr>
<td>not (all are p).</td>
<td>Some are not p.</td>
</tr>
<tr>
<td>not (some are p).</td>
<td>None are p.</td>
</tr>
</tbody>
</table>

Use the following reasoning patterns to determine which logical conclusion is or is not warranted:

**Pattern I:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>p.</td>
</tr>
<tr>
<td>conclusion</td>
<td></td>
<td>q.</td>
</tr>
</tbody>
</table>

**Pattern II:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>conclusion</td>
<td>not q.</td>
</tr>
</tbody>
</table>

**Pattern III:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>conclusion</td>
<td>not p.</td>
</tr>
</tbody>
</table>

**Pattern IV:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
<th>If p, then q.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>q.</td>
</tr>
<tr>
<td>conclusion</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Pattern V:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
<th>p or q.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>not q.</td>
</tr>
<tr>
<td>conclusion</td>
<td></td>
<td>p.</td>
</tr>
</tbody>
</table>

**Pattern VI:**

<table>
<thead>
<tr>
<th>premise 1</th>
<th>premise 2</th>
<th>p and q.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>p.</td>
</tr>
<tr>
<td>conclusion</td>
<td></td>
<td>q.</td>
</tr>
</tbody>
</table>

354
Skill IV E1  Draw logical conclusions when facts warrant them

In drawing logical conclusions, it may be helpful to use the following steps:

**Step 1.** Choose a letter to represent each statement.

**Step 2.** Number each premise and express it using the letters from Step 1.

**Step 3.** Use the premises and the logical reasoning pattern to draw a logical conclusion if one exists.

**Reminder:** It may help to symbolize each possible conclusion given in the response options using the letters chosen. When a letter (p) symbolizes a negative statement (it does not rain), the negation (not p) is a positive statement (it does rain).

**Examples**

1. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.

   If you go to a picnic, then you have a great time. If you have a great time, then you feel good. You go to a picnic.

   A. You don’t have a great time.
   B. You don’t feel good.
   C. You feel good.
   D. None of the above is warranted.

   **Step 1.** Choose a letter to represent each statement.

   p: You go to a picnic.
   q: You have a great time.
   r: You feel good.

   If it helps, also symbolize each response option, using the letters chosen.

   A. not q.
   B. not r.
   C. r.

   **Step 2.** Number each premise and express it using the letters from Step 1.

   1. If p (you go to a picnic), then q (you have a great time).
   2. If q (you have a great time), then r (you feel good).
   3. p (you go to a picnic).


Skill IVE1  

*Draw logical conclusions when facts warrant them*

**Step 3.** Use the premises and the logical reasoning pattern to draw a logical conclusion if one exists.

Apply Pattern IV to premise 1 and premise 2:

- Premise 1: If \( p \), then \( q \). If \( p \) (you go to a picnic), then \( q \) (you have a great time).
- Premise 2: If \( q \), then \( r \). If \( q \) (you have a great time), then \( r \) (you feel good).
- Conclusion 1: If \( p \), then \( r \). If \( p \) (you go to a picnic), then \( r \) (you feel good).

Apply Pattern I to conclusion 1 and premise 3:

- Conclusion 1: If \( p \), then \( r \). If \( p \) (you go to a picnic), then \( r \) (you feel good).
- Premise 3: \( p \) (you go to a picnic).
- Conclusion 2: \( r \) (you feel good).

Therefore, option C, "You feel good," is the correct answer.

2. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.

Joe plays golf or Sam plays tennis. If Joe plays golf, then he owns golf clubs. Sam does not play tennis.

A. Joe does not play golf.
B. Joe owns golf clubs.
C. Joe does not own golf clubs.
D. None of the above is warranted.

**Step 1.** Choose a letter to represent each statement.

- \( p \): Joe plays golf.
- \( q \): Sam plays tennis.
- \( r \): Joe owns golf clubs.

If it helps, also symbolize each response option, using the letters chosen.

A. not \( p \).
B. \( r \).
C. not \( r \).

**Step 2.** Number each premise and express it using the letters from Step 1.

1. \( p \) (Joe plays golf) or \( q \) (Sam plays tennis).
2. If \( p \) (Joe plays golf), then \( r \) (Joe owns golf clubs).
3. not \( q \) (Sam plays tennis).

\( p \) or \( q \).
If \( p \), then \( r \).
not \( q \).
Skill IVE1  

**Draw logical conclusions when facts warrant them**

3. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.

If Joe is a music major, he plays the piano. If he plays the piano, he plays the organ. Joe plays the organ.

A. Joe is not a music major.
B. Joe does not play the piano.
C. Joe is a music major.
D. None of the above is warranted.

**Step 1.** Choose a letter to represent each statement.

- p: Joe is a music major.
- q: Joe plays the piano.
- r: Joe plays the organ.

If it helps, also symbolize each response option, using the letters chosen.

A. not p.
B. not q.
C. p.

**Step 2.** Number each premise and express it using the letters from Step 1.

1. If p (Joe is a music major), then q (Joe plays the piano). If p, then q.
2. If q (Joe plays the piano), then r (Joe plays the organ). If q, then r.
3. r (Joe plays the organ). r.
Skill IVE1  

**Draw logical conclusions when facts warrant them**

**Step 3.** Use the premises and the logical reasoning pattern to draw a logical conclusion if one exists.

Apply Pattern IV to premise 1 and premise 2:

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>If p, then q.</th>
<th>If p (Joe is a music major), then q (Joe plays the piano).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>If q, then r.</td>
<td>If q (Joe plays the piano), then r (Joe plays the organ).</td>
</tr>
<tr>
<td>Conclusion</td>
<td>If p, then r.</td>
<td>If p (Joe is a music major), then r (Joe plays the organ).</td>
</tr>
</tbody>
</table>

Premise 3 (r) does not allow for drawing a conclusion. Therefore, option D, “None of the above is warranted,” is the correct answer.

4. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.

If it rains, then the ball game will not be played. If the ball game is not played, then the boys will go to a movie. The boys go to the mall or it rained. They didn’t go to the mall.

A. It does not rain.
B. The ball game will be played.
C. The boys go to a movie.
D. None of the above is warranted.

**Step 1.** Choose a letter to represent each statement.

| p: It rains. |
| q: The ball game will not be played. |
| r: The boys will go to a movie. |
| s: The boys go to the mall. |

If it helps, also symbolize each response option, using the letters chosen.

A. not p.
B. not q.
C. r.

**Step 2.** Number each premise and express it using the letters from Step 1.

1. If p (It rains), then q (the ball game will not be played).  
2. If q (the ball game is not played), then r (the boys will go to a movie).  
3. s (the boys go to the mall) or p (it rains).  
4. not s (The boys go to the mall).
Skill IVE1  

*Draw logical conclusions when facts warrant them*

**Step 3.** Use the premises and the logical reasoning pattern to draw a logical conclusion if one exists.

Apply Pattern IV to premise 1 and premise 2:

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>If p, then q.</th>
<th>If p (it rains), then q (the ball game will not be played).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>If q, then r.</td>
<td>If q (the ball game will not be played), then r (the boys will go to a movie).</td>
</tr>
<tr>
<td>Conclusion 1</td>
<td>If p, then r.</td>
<td>If p (it rains), then r (the boys will go to a movie).</td>
</tr>
</tbody>
</table>

Apply Pattern III to premise 3 and premise 4:

<table>
<thead>
<tr>
<th>Premise 3</th>
<th>s or p.</th>
<th>s (the boys go to the mall) or p (it rains).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 4</td>
<td>not s.</td>
<td>not s (the boys go to the mall).</td>
</tr>
<tr>
<td>Conclusion 2</td>
<td>p.</td>
<td>p (it rains).</td>
</tr>
</tbody>
</table>

Apply Pattern IV to conclusion 2 and conclusion 1:

<table>
<thead>
<tr>
<th>Conclusion 1</th>
<th>If p, then r.</th>
<th>If p (it rains), then r (the boys will go to a movie).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conclusion 2</td>
<td>p.</td>
<td>p (it rains).</td>
</tr>
<tr>
<td>Conclusion 3</td>
<td>r.</td>
<td>r (the boys will go to a movie).</td>
</tr>
</tbody>
</table>

Therefore, option C, "The boys go to a movie," is the correct answer.
# LOGIC

## PRACTICE PROBLEMS

<table>
<thead>
<tr>
<th>SKILLS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>IE1</td>
<td>Deduce facts of set inclusion and noninclusion from diagrams</td>
</tr>
<tr>
<td>IIE1</td>
<td>Identify negations of simple and compound statements</td>
</tr>
<tr>
<td>IIE2</td>
<td>Determine equivalence or nonequivalence of statements</td>
</tr>
<tr>
<td>IIIE2</td>
<td>Identify rules for transforming statements</td>
</tr>
<tr>
<td>IIE3</td>
<td>Draw logical conclusions from data</td>
</tr>
<tr>
<td>IIE4</td>
<td>Identify invalid arguments that have true conclusions</td>
</tr>
<tr>
<td>IIIE1</td>
<td>Identify valid reasoning patterns</td>
</tr>
<tr>
<td>IVE1</td>
<td>Draw logical conclusions when facts warrant them</td>
</tr>
</tbody>
</table>
1. Sets A, B, C, and U are related as shown in the diagram.

Which of the following statements is true, assuming not one of the four regions is empty?

A. Any element that is a member of set A is also a member of set B.
B. No element is a member of all three sets: A, B, and C.
C. Any element that is a member of set U is also a member of set A.
D. None of the above statements is true.

2. Sets A, B, C, and U are related as shown in the diagram.

Which of the following statements is true, assuming not one of the six regions is empty?

A. Any element that is a member of set B is also a member of set A.
B. No element is a member of all three sets: A, B, and C.
C. Any element that is a member of set U is also a member of set B.
D. None of the above statements is true.
### Logic Practice Problems

#### IDENTIFY NEGATIONS OF SIMPLE AND COMPOUND STATEMENTS

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.</strong> Select the statement that is the negation of the statement “All winter days are cold.”</td>
<td><strong>5.</strong> Select the statement that is the negation of the statement “If the weather is cold, then the ball game will not be played.”</td>
</tr>
<tr>
<td>A. Some winter days are cold.</td>
<td>A. If the weather is not cold, then the ball game will be played.</td>
</tr>
<tr>
<td>B. Some winter days are not cold.</td>
<td>B. The weather is cold and the ball game was not played.</td>
</tr>
<tr>
<td>C. No winter days are cold.</td>
<td>C. If the ball game is played, then the weather is not cold.</td>
</tr>
<tr>
<td>D. If it is not a winter day, then it is not cold.</td>
<td>D. The weather is cold and the ball game will be played.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4.</strong> Select the statement that is the negation of the statement “The sun is shining or the store is closed.”</td>
<td><strong>6.</strong> Select the statement that is the negation of the statement “No cats are felines.”</td>
</tr>
<tr>
<td>A. The sun is not shining or the store is not closed.</td>
<td>A. Some cats are felines.</td>
</tr>
<tr>
<td>B. The sun is shining and the store is not closed.</td>
<td>B. Some cats are not felines.</td>
</tr>
<tr>
<td>C. The sun is not shining and the store is not closed.</td>
<td>C. If an animal is a feline, then it is not a cat.</td>
</tr>
<tr>
<td>D. If the sun is shining, then the store is not closed.</td>
<td>D. All cats are felines.</td>
</tr>
</tbody>
</table>
### Logic Practice Problems

**DETERMINE EQUIVALENCE OR NONEQUIVALENCE OF STATEMENTS**

<table>
<thead>
<tr>
<th>7.</th>
<th>Select the statement below that is logically equivalent to “If Tom studies, then he will pass CLAST.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>If Tom does not study, then he will not pass CLAST.</td>
</tr>
<tr>
<td>B.</td>
<td>If Tom passed CLAST, then he studied.</td>
</tr>
<tr>
<td>C.</td>
<td>If Tom did not pass CLAST, then he did not study.</td>
</tr>
<tr>
<td>D.</td>
<td>Tom studies and does not pass CLAST.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9.</th>
<th>Select the statement below that is NOT logically equivalent to “If Mary works late, then Joe will prepare dinner.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>If Joe prepares dinner, then Mary works late.</td>
</tr>
<tr>
<td>B.</td>
<td>If Joe does not prepare dinner, then Mary did not work late.</td>
</tr>
<tr>
<td>C.</td>
<td>Joe prepares dinner or Mary does not work late.</td>
</tr>
<tr>
<td>D.</td>
<td>Mary does not work late or Joe prepares dinner.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.</th>
<th>Select the statement below that is logically equivalent to “It is not true that Jim is playing golf or Mary is playing tennis.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>Jim is not playing golf or Mary is not playing tennis.</td>
</tr>
<tr>
<td>B.</td>
<td>Jim is playing golf and Mary is not playing tennis.</td>
</tr>
<tr>
<td>C.</td>
<td>If Jim is not playing golf, then Mary is not playing tennis.</td>
</tr>
<tr>
<td>D.</td>
<td>Jim is not playing golf and Mary is not playing tennis.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10.</th>
<th>Select the statement below that is logically equivalent to “It is not true that some dogs bark or some birds do not have feathers.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>Some dogs do not bark and some birds have feathers.</td>
</tr>
<tr>
<td>B.</td>
<td>No dogs bark and all birds have feathers.</td>
</tr>
<tr>
<td>C.</td>
<td>Some dogs do not bark or some birds have feathers.</td>
</tr>
<tr>
<td>D.</td>
<td>No dogs bark and some birds have feathers.</td>
</tr>
</tbody>
</table>
Logic Practice Problems

IDENTIFY RULES FOR TRANSFORMING STATEMENTS

11. Select the rule of logical equivalence that directly (in one step) transforms statement “i” into statement “ii.”

i. If Joe takes calculus, then he will buy a calculator.

ii. Joe will not take calculus or he will buy a calculator.

A. “If p, then q” is equivalent to “if not q, then not p.”

B. “If p, then q” is equivalent to “(not p) or q.”

C. “Not (p and q)” is equivalent to “(not p) or (not q).”

D. Correct equivalence rule is not given.

12. Select the rule of logical equivalence that directly (in one step) transforms statement “i” into statement “ii.”

i. Not all of the students have calculators.

ii. Some students do not have calculators.

A. “If p, then q” is equivalent to “if not q, then not p.”

B. “All are not p” is equivalent to “none are p.”

C. “Not (not p)” is equivalent to “p.”

D. “Not all are p” is equivalent to “some are not p.”
13. Read the requirements and each applicant’s qualifications for the opportunity to buy a condominium at the beach.

To qualify to buy the condominium an applicant must have a gross income of $50,000 if single ($65,000 combined income if married) and have no pets.

Mr. Blue is single and makes $55,000. He has a cat.

Mrs. Green and her husband have no pets. They both work. Mr. Green makes $32,000 and Mrs. Green makes $35,000.

Mr. Pao is married. He makes $60,000. His wife has no income. They have no pets.

Identify which of the applicants would qualify for the opportunity to buy the condominium.

A. Mr. Blue
B. Mrs. Green
C. Mr. Pao
D. None of the above

14. Given that:

i. No athletes are lazy.
ii. All basketball players are athletes.

determine which conclusion can be logically deduced.

A. No basketball player is lazy.
B. All basketball players are lazy.
C. Some basketball players are lazy.
D. None of the above is true.
<table>
<thead>
<tr>
<th></th>
<th>15. All of the following arguments A–D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>All birds have wings and all robins are birds; therefore, all robins have wings.</td>
</tr>
<tr>
<td>B</td>
<td>All robins have wings and all birds have wings; therefore, all robins are birds.</td>
</tr>
<tr>
<td>C</td>
<td>All turtles are reptiles and all reptiles have a scaly skin; therefore, turtles have a scaly skin.</td>
</tr>
<tr>
<td>D</td>
<td>All mammals have hair. A deer is a mammal. Therefore, a deer has hair.</td>
</tr>
<tr>
<td></td>
<td>16. All of the following arguments A–D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.</td>
</tr>
<tr>
<td>A</td>
<td>All spiders are predaceous. The black widow is predaceous. Therefore, the black widow is a spider.</td>
</tr>
<tr>
<td>B</td>
<td>All sea stars are echinoderms and all echinoderms are marine; therefore, all sea stars are marine.</td>
</tr>
<tr>
<td>C</td>
<td>All frogs are amphibians and all amphibians breathe by lungs, gills, or skin; therefore, all frogs breathe by lungs, gills, or skin.</td>
</tr>
<tr>
<td>D</td>
<td>All mammals have hair and all giraffes are mammals; therefore, all giraffes have hair.</td>
</tr>
<tr>
<td>17.</td>
<td>Select the conclusion that will make the following argument valid.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>If I pass the CLAST, then I will get my AA degree. If I get my AA degree, then I will attend the university.</td>
</tr>
<tr>
<td>A.</td>
<td>If I do not pass the CLAST, then I will not attend the university.</td>
</tr>
<tr>
<td>B.</td>
<td>If I get my AA degree, then I pass the CLAST.</td>
</tr>
<tr>
<td>C.</td>
<td>If I pass the CLAST, then I will attend the university.</td>
</tr>
<tr>
<td>D.</td>
<td>If I pass the CLAST, then I will not attend the university.</td>
</tr>
<tr>
<td>19.</td>
<td>Select the conclusion that will make the following argument valid.</td>
</tr>
<tr>
<td></td>
<td>Some people vote or all issues pass. No people vote.</td>
</tr>
<tr>
<td>A.</td>
<td>Some issues did not pass.</td>
</tr>
<tr>
<td>B.</td>
<td>All issues pass.</td>
</tr>
<tr>
<td>C.</td>
<td>All people do not vote and some issues do not pass.</td>
</tr>
<tr>
<td>D.</td>
<td>No people vote and some issues did not pass.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>18.</th>
<th>Select the conclusion that will make the following argument valid.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>If I am tired, then I need more sleep. I do not need more sleep.</td>
</tr>
<tr>
<td>A.</td>
<td>If I need more sleep, then I am tired.</td>
</tr>
<tr>
<td>B.</td>
<td>If I am not tired, then I do not need more sleep.</td>
</tr>
<tr>
<td>C.</td>
<td>I am tired.</td>
</tr>
<tr>
<td>D.</td>
<td>I am not tired.</td>
</tr>
</tbody>
</table>
### Logic Practice Problems

#### Skill IVE1

**DRAW LOGICAL CONCLUSIONS WHEN FACTS WARRANT THEM**

| 20. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.  
If I pass this test, then I will graduate. I pass this test or I get a job. I did not get a job.  
A. I did not pass this test.  
B. I did not graduate.  
C. I did graduate.  
D. None of the above is warranted. |
|---|
| 22. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.  
Mary eats ice cream or she eats yogurt. If Mary eats yogurt, then she is healthy. If Mary is healthy, then she can run the marathon. Mary does not eat yogurt.  
A. Mary does not eat ice cream.  
B. Mary is healthy.  
C. If Mary runs the marathon, then she eats yogurt.  
D. None of the above is warranted. |

| 21. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select option D.  
If all cars have cruise control, then no person speeds. If no person speeds, then no tickets are given. Some tickets are given.  
A. No person speeds.  
B. Some cars do not have cruise control.  
C. All cars have cruise control.  
D. None of the above is warranted. |
# LOGIC PRACTICE EXPLANATIONS

<table>
<thead>
<tr>
<th>SKILLS</th>
<th>PROBLEMS</th>
</tr>
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<td>Identify valid reasoning patterns</td>
</tr>
<tr>
<td>IVE1</td>
<td>Draw logical conclusions when facts warrant them</td>
</tr>
</tbody>
</table>
DEDUCE FACTS OF SET INCLUSION AND NONINCLUSION FROM DIAGRAMS

1. \( B \) is the correct response. In \( A \), the statement is incorrect because set \( B \) does not completely overlap set \( A \). In \( C \), the statement is incorrect because set \( A \) does not completely overlap set \( U \). In \( D \), the answer is not applicable.

2. \( D \) is the correct response. In \( A \), the statement is incorrect because set \( A \) does not completely overlap set \( B \). In \( B \), the statement is incorrect because sets \( A \), \( B \), and \( C \) have a common overlap. Those elements in sets \( A \) and \( C \) are also members of set \( B \). In \( C \), the statement is incorrect because set \( B \) does not completely overlap set \( U \). Any element that is a member of set \( B \) is also a member of set \( U \).

IDENTIFY NEGATIONS OF SIMPLE AND COMPOUND STATEMENTS

3. \( B \) is the correct response. In \( A \), the negation of this statement would be “No winter days are cold.” In \( C \), “no” is the negation of “some are.” In \( D \), the statement is not a negation of the original statement.

4. \( C \) is the correct response. In \( A \), the negation of an or statement cannot be an or statement. It is an and statement. In \( B \), the negation of an or statement must negate each statement in the compound sentence. In \( D \), an if, then statement is not the negation of any compound statement. Also, the negation of an or statement contains the connective and.

5. \( D \) is the correct response. In \( A \), the negation of an if, then statement is never an if, then statement. In \( B \), the negation of an if, then statement is an and statement but the statement following the then statement in the compound statement must be negated. In \( C \), the negation of an if, then statement will never be an if, then statement.

6. \( A \) is the correct response. In \( B \), the negation of a statement containing the quantifier “no” is “some” and not “some—not.” In \( C \), the negation of a statement containing the quantifier “no” is “some” and not an if, then statement. In \( D \), the negation of a statement containing the quantifier “no” is “some” and not an all statement.
DETERMINE EQUIVALENCE OR NONEQUIVALENCE OF STATEMENTS

7. \( C \) is the correct response. In \( A \), the answer is the inverse “if not \( p \), then not \( q \)” “In \( B \), the answer is the converse “if \( q \), then \( p \)” “In \( D \), the answer is the negation. Remember that the converse, inverse, and negation are not the equivalent to the original statement.

8. \( D \) is the correct response. “Not (\( p \) or \( q \))” is logically equivalent to “(not \( p \)) and (not \( q \)).” Responses \( A, B, \) and \( C \) are not logically equivalent to “not (\( p \) or \( q \)).”

9. \( A \) is the correct response. In \( B \), the answer is the form “if (not \( q \)), then (not \( p \)).” In \( C \), the answer is the form “(q) or (not \( p \)).” In \( D \), the answer is the form “(not \( p \)) or (q).”

10. \( B \) is the correct response because “Not [(some are \( p \)) or (some are not \( q \))]” is logically equivalent to “(none are \( p \)) and (all are \( q \)).” In \( A \) and \( C \), the responses are incorrect because “Some dogs do not bark” is not logically equivalent to “It is not true that some dogs bark” and “Some birds have feathers” is not logically equivalent to “It is not true that some birds do not have feathers.” In \( D \), “Some birds have feathers” is not logically equivalent to “It is not true that some birds do not have feathers.”

IDENTIFY RULES FOR TRANSFORMING STATEMENTS

11. \( B \) is the correct response. In \( A \), statement “\( i \)ii” is not in the form “if not \( q \), then not \( p \)” “In \( C \), statement “\( i \)” is in the form “if \( p \), then \( q \)” “Statement “\( ii \)” is in the form “(not \( p \)) or \( q \)” “In \( D \), this option is obviously not correct because \( B \) is the correct rule.

12. \( D \) is the correct response. In \( A \), statement “\( i \)” is not in the form “if \( p \), then \( q \)” “In \( B \), statement “\( i \)” is not in the form “all are not \( p \)” “In \( C \), statement “\( i \)” is not in the form “not (not \( p \))” “In \( D \), statement “\( i \)” is in the form “not all are \( p \)” “Statement “\( ii \)” is in the form “some are not \( p \)”
DRAW LOGICAL CONCLUSIONS FROM DATA

13. **B** is the correct response. In **A**, Mr. Blue has a cat. The stipulation was that there were no pets. His salary is adequate. In **C**, the salary is below the requirement since he is married. In **D**, the response is not accurate because Mrs. Green qualifies.

14. **A** is the correct response because

If p, then not q. If a person is an athlete (p), then he or she is not lazy (not q).
If r, then p. If a person is a basketball player (r), then he or she is an athlete (p).
If r, then not q. If a person is a basketball player (r), then he or she is not lazy (not q).

In **B**, the statement is incorrect because basketball players are contained within the set of athletes, and the sets “athletes” and “lazy” are completely separate. In **C**, the statement is incorrect because basketball players are completely contained in the set of athletes, and the sets “athletes” and “lazy” are completely separate. Therefore, the sets “lazy” and “basketball” have nothing in common. In **D**, the response is not accurate because **A** applies.
IDENTIFY INVALID ARGUMENTS THAT HAVE TRUE CONCLUSIONS

15. B is the correct response. The argument is not valid because all of these are possibilities:

![Diagrams showing different relationships between birds, robins, wings, scaly skin, reptiles, turtles, deer, and mammals.]

You are not forced to accept that all robins are birds.

In A, the argument may be diagrammed:

In C, the argument may be diagrammed:

In D, the argument may be diagrammed:
16. A is the correct response. The argument is not valid because all of these are possibilities:

You are not forced to accept that the black widow is a spider.

In B, the argument may be diagrammed:

In C, the argument may be diagrammed:

In D, the argument may be diagrammed:
IDENTIFY VALID REASONING PATTERNS

17. **C** is the correct response because the argument is in the form of

\[
\text{If } p, \text{ then } q. \quad \text{If } p \text{ (I pass the CLAST), then } q \text{ (I get my AA degree).} \\
\text{If } q, \text{ then } r. \quad \text{If } q \text{ (I get my AA degree), then } r \text{ (I go to the university).} \\
\hline
\text{conclusion } \text{If } p, \text{ then } r. \quad \text{If } p \text{ (I pass the CLAST), then } r \text{ (I go to the university).}
\]

In **A**, the conclusion is "If not } p, \text{ then not } r," which is the inverse of the answer. In **B**, the conclusion is "If } q, \text{ then } p," which is the converse of the first premise. In **D**, the conclusion is "If } p, \text{ then not } r."

18. **D** is the correct response because the argument is in the form of

\[
\text{If } p, \text{ then } q. \quad \text{If } p \text{ (I am tired), then } q \text{ (I need more sleep).} \\
\text{not } q. \quad \text{not } q \text{ (I need more sleep).} \\
\hline
\text{conclusion } \text{not } p. \quad \text{not } p \text{ (I am tired).}
\]

In **A**, the conclusion is "If } q, \text{ then } p," which is the converse of the first premise. In **B**, the conclusion is "If not } p, \text{ then not } q," which is the inverse of the first premise. In **C**, the conclusion is "p," which is the negation of the correct response.

19. **B** is the correct response because the argument is in the form of

\[
\text{p or } q. \quad \text{p (some people vote) or } q \text{ (all issues pass).} \\
\text{not } p. \quad \text{not } p \text{ (some people vote).} \\
\hline
\text{conclusion } q. \quad q \text{ (all issues pass).}
\]

In **A**, the conclusion is "not } q," which is the negation of the correct response. In **C**, the conclusion is "not } p \text{ and not } q," which is a negation of the first premise. In **D**, the conclusion is also "not } p \text{ and not } q," another form of the negation of the first premise.
DRAW LOGICAL CONCLUSIONS WHEN FACTS WARRANT THEM

20. C is the correct response because the argument follows two logical reasoning patterns:

<table>
<thead>
<tr>
<th>Premise 2</th>
<th>p or r.</th>
<th>p (I pass the test) or r (I get the job).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 3</td>
<td>not r.</td>
<td>not r (I did not get the job).</td>
</tr>
<tr>
<td>Conclusion 1</td>
<td>p.</td>
<td>p (I pass the test).</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>If p, then q.</th>
<th>If p (I pass the test), then q (I will graduate).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conclusion 1</td>
<td>p.</td>
<td>p (I pass the test).</td>
</tr>
<tr>
<td>Conclusion 2</td>
<td>q.</td>
<td>q (I will graduate).</td>
</tr>
</tbody>
</table>

In A, the conclusion is “not p,” which does not follow logical reasoning patterns. In B, the conclusion is “not q,” which is the negation of the correct response. In D, the response is incorrect because C is the correct response.

21. B is the correct response because the argument follows two logical reasoning patterns:

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>If p, then q.</th>
<th>If p (all cars have cruise control), then q (no person speeds).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>If q, then r.</td>
<td>If q (no person speeds), then r (no tickets are given).</td>
</tr>
<tr>
<td>Conclusion 1</td>
<td>If p, then r.</td>
<td>If p (all cars have cruise control), then r (no tickets are given).</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>Conclusion 1</th>
<th>If p, then r.</th>
<th>If p (all cars have cruise control), then r (no tickets are given).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 3</td>
<td>not r.</td>
<td>not r (no tickets are given).</td>
</tr>
<tr>
<td>Conclusion 2</td>
<td>not p.</td>
<td>not p (all cars have cruise control).</td>
</tr>
</tbody>
</table>

In A, the conclusion is “q,” which does not follow logical reasoning patterns. In C, the conclusion is “p,” which is the negation of the conclusion. In D, the response is incorrect because B is the correct response.
22. *D* is the correct response because none of the other options is a correct conclusion from logical reasoning patterns. In *A*, the logical conclusion following the first and fourth premise would be “Mary eats ice cream.”

| premise 1 | p or q. | p (Mary eats ice cream) or q (Mary eats yogurt). |
| premise 4 | not q.  | not q (Mary does not eat yogurt).               |
| conclusion A | p. | p (Mary eats ice cream).                       |

In *B*, the logical reasoning pattern was used incorrectly:

| premise 2 | If q, then r. | If q (Mary eats yogurt), then r (Mary is healthy). |
| premise 4 | not q.        | *This has to be q to reach conclusion B.*        |
| conclusion B | r. | r (Mary is healthy) is not a valid conclusion.  |

In *C*, this conclusion is the converse of a possible correct conclusion.