MAC 2233
Spring 2011

EXAM 1A

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 1, 1.
   • 2 3 4 5 6 7 8 9 0
   • 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code”, encode A.
   • B C D E

E. 1) This test consists of 4 three-point and 9 five-point multiple choice questions, one
   three-point bonus question, and two pages (both sides) of free response questions
   worth 25 points.
   2) The time allowed is 90 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper or if you have a problem with your
   test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE
   TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:
   1) Before turning in your test, check for transcribing errors. Any mistakes you leave in
      are there to stay.
   2) Bring your test, scratch paper, bubble sheet, and any tearoff sheets to your proctor
      to turn them in. Be prepared to show your UF ID card.
   3) Answers will be posted in Sakai after the test.
1. What is the domain of the function \( f(x) = \sqrt{\frac{1-x^2}{x}} \)

A. \((0, 1]\)  
B. \([-1, 0) \cup [1, \infty)\)  
C. \((-\infty, 1]\)  
D. \((-\infty, -1] \cup (0, 1]\)  
E. \((0, \infty)\)

2. If \( f(x) = x^3 - 9x \), then \( f'(x) = 3x^2 - 9 \). Find each value of \( x \) at which the graph of \( f(x) \) has a horizontal tangent line.

A. \( x = -9 \) only  
B. \( x = 0 \) only  
C. \( x = 0 \) and \( x = -9 \)  
D. \( x = \sqrt{3} \) only  
E. \( x = -\sqrt{3} \) and \( x = \sqrt{3} \)

3. If \( f(x) = \sqrt{x} \) and \( g(x) = \frac{x+5}{x-1} \), find \( (f \circ g)(x) \) and its domain.

A. \( (f \circ g)(x) = \frac{\sqrt{x}+5}{\sqrt{x}-1} \) \([0, 1) \cup (1, \infty)\)

B. \( (f \circ g)(x) = \frac{\sqrt{x}+5}{\sqrt{x}-1} \) \([0, \infty)\)

C. \( (f \circ g)(x) = \sqrt{\frac{x+5}{x-1}} \) \([0, 1) \cup (1, \infty)\)

D. \( (f \circ g)(x) = \sqrt{\frac{x+5}{x-1}} \) \((-\infty, -5] \cup (1, \infty)\)

E. \( (f \circ g)(x) = \frac{\sqrt{x}+5}{x-1} \) \([-5, 1) \cup (1, \infty)\)
4. Let \( f(x) = \begin{cases} \frac{x^2 - 5x + 6}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases} \). Evaluate \( \lim_{{x \to 2}} f(x) \).

A. \(-3\)  
B. \(4\)  
C. \(2\)  
D. \((-1)\)  
E. Does Not Exist

Questions 6 - 14 are worth 5 points each.

5. If \( f(x) = \frac{\sqrt{2x+1} - \sqrt{3}}{x - 1} \), find \( \lim_{{x \to 1}} f(x) \).

A. \(\frac{2}{\sqrt{3}}\)  
B. \(2\)  
C. \(\frac{1}{2\sqrt{3}}\)  
D. \(\frac{1}{3}\)  
E. \(\frac{1}{\sqrt{3}}\)

6. The supply and demand functions for a certain product are \( p = x^2 + 9x + 40 \) and \( p = 70 - 2x^2 \) respectively, where \( p \) is the price of the product and \( x \) is the number of items measured in hundreds. Find the equilibrium quantity and price.

A. 200 units; $38  
B. 200 units; $62  
C. 500 units; $20  
D. 500 units; $110  
E. 300 units; $52

7. The Intermediate Value Theorem guarantees that the equation \( x^3 + 2x^2 - 2 = 0 \) has a solution on which of the following intervals?

A. \((-2, -1)\)  
B. \((-1, 0)\)  
C. \((1, 2)\)  
D. \((0, 1)\)  
E. \((-2, 0)\)

8. Find the value of \( k \) so that the line passing through the points \((4, 2k + 9)\) and \((k, 2)\) is perpendicular to the line \( x + 3y - 6 = 0 \).

A. \(-\frac{2}{3}\)  
B. \(1\)  
C. \(-5\)  
D. \(19\)  
E. \(-\frac{17}{7}\)
9. If \( f(x) = |x| \), which function below best represents the given graph?

\[ \begin{align*}
\text{A. } y &= 2 - f(x - 3) \\
\text{B. } y &= 2 - f(x + 3) \\
\text{C. } y &= f(x + 3) - 2 \\
\text{D. } y &= f(x - 3) + 2 \\
\text{E. } y &= 3 - f(x + 2)
\end{align*} \]

10. Let \( p = \lim_{x \to \infty} \frac{5 - 3x^4}{6 - 3x + x^3} \) and \( q = \lim_{x \to \infty} \frac{1 - x^3}{(x^2 - 1)(3x + 1)} \). Find \( p \) and \( q \).

\[ \begin{align*}
\text{A. } p &= -\infty \text{ and } q = -1/3 \\
\text{B. } p &= +\infty \text{ and } q = -1/3 \\
\text{C. } p &= -\infty \text{ and } q = -3 \\
\text{D. } p &= -3 \text{ and } q = -3 \\
\text{E. } p &= 5/6 \text{ and } q = -3
\end{align*} \]

11. Evaluate \( \lim_{x \to 2^+} \frac{6 - 3x}{|2 - x|} \)

\[ \begin{align*}
\text{A. } 3 \\
\text{B. } 3/2 \\
\text{C. } -3/2 \\
\text{D. } -3 \\
\text{E. The limit does not exist.}
\end{align*} \]
12. Write the equation of the circle with center $(4, -2)$ and containing the point $(0, 1)$.

A. $(x - 4)^2 + (y + 2)^2 = 5$
B. $x^2 + (y - 1)^2 = 25$
C. $(x - 4)^2 + (y + 2)^2 = 25$
D. $(x + 4)^2 + (y - 2)^2 = 5$
E. $(x + 2)^2 + (y - 4)^2 = 25$

13. Which of the following statements is/are true of the function below?

$$f(x) = \begin{cases} 
5 & \text{if } x < 1 \\
1 + \frac{4}{x} & \text{if } 1 < x \leq 4 \\
\sqrt{x} & \text{if } x > 4 
\end{cases}$$

I. $f(x)$ has one removable discontinuity.
II. $f(x)$ has one nonremovable discontinuity.
III. $f(x)$ is continuous at $x = 4$.

A. I only          B. III only          C. I & II only
D. II & III only   E. I & III only
1. Let $f(x) = x^2 - 2x + 3$.
   
   (a) Use the limit definition of derivative to find $f'(x)$.

   (b) Write the equation of the tangent line to $y = f(x)$ at $x = 0$. Write you answer in slope-intercept form.
2. Let

\[ f(x) = \begin{cases} 
  |x| + 1 & \text{if } x < 0 \\ 
  1 & \text{if } 0 < x < 3 \\ 
  \frac{-1}{x-3} & \text{if } x > 3 
\end{cases} \]

(a) Evaluate the following limits; use symbols $\infty, -\infty$ or D.N.E. when appropriate:

\[ \lim_{x \to 0^-} f(x) = \ldots \quad \lim_{x \to 0^+} f(x) = \ldots \quad \lim_{x \to 0} f(x) = \ldots \]

\[ \lim_{x \to 3^-} f(x) = \ldots \quad \lim_{x \to 3^+} f(x) = \ldots \quad \lim_{x \to 3} f(x) = \ldots \]

(b) Sketch the graph of $f(x)$ on the axes below.

(c) List each value of $x$ at which $f(x)$ is discontinuous, and describe each as removable, or nonremovable. If the discontinuity is removable, state how you can define or redefine $f(x)$ to make the function continuous there.
3. A manufacturer has a monthly fixed cost of $40,000 and a production cost of $8 for each unit produced. The product sells for $12 per unit. Let \( x \) denote the number of units.

(a) What is the cost function?

(b) What is the revenue function?

(c) What is the profit function?

(d) What is the break-even point?

(e) How many units must be sold to make a profit of 80,000 dollars?
4. While reviewing his accounts over the past year, the manager of a mall T-shirt stand made the following observations. During a normal week, he sold an average of 40 Gator T-shirts at a price $18 each. When he reduced the price by $6 for a clearance sale, an average of 10 more T-shirts sold per week.

(a) Find a linear model expressing demand \( x \), the average number of T-shirts sold weekly as function of \( p \), the price of a T-shirt.

\[ x(p) = \text{______________} \]

(b) One week the manager sold 58 T-shirts. Use your function to find the selling price of a T-shirt that week.

\[ $\text{______________} \]

(c) Find the corresponding revenue function where revenue is written as a function of price \( p \).

\[ R(p) = \text{______________} \]
MAC 2233
Spring 2011

EXAM 1B

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:
   1) Name (last name, first initial, middle initial)
   2) UF ID Number
   3) Section Number

C. Under “special codes”, code in the test ID number 1, 2.
   ○ 2 3 4 5 6 7 8 9 0
   1 ○ 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code”, encode B.
   A ○ C D E

E. 1) This test consists of 4 three-point and 9 five-point multiple choice questions, one
   three-point bonus question, and two pages (both sides) of free response questions
   worth 25 points.
   2) The time allowed is 90 minutes.
   3) You may write on the test.
   4) Raise your hand if you need more scratch paper or if you have a problem with your
   test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE
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F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

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MAC 2233 — Spring 2011 — EXAM 1B

Questions 1 - 5 are worth 3 points each.

1. If \( f(x) = \sqrt{x} \) and \( g(x) = \frac{x+2}{x-5} \), find \((f \circ g)(x)\) and its domain.

   A. \( (f \circ g)(x) = \frac{\sqrt{x} + 2}{\sqrt{x} - 5} \) \( [0, 25) \cup (25, \infty) \)

   B. \( (f \circ g)(x) = \frac{\sqrt{x} + 2}{\sqrt{x} - 5} \) \( [0, \infty) \)

   C. \( (f \circ g)(x) = \sqrt{\frac{x + 2}{x - 5}} \) \( [0, 5) \cup (5, \infty) \)

   D. \( (f \circ g)(x) = \sqrt{\frac{x + 2}{x - 5}} \) \( (-\infty, -2] \cup (5, \infty) \)

   E. \( (f \circ g)(x) = \frac{\sqrt{x} + 2}{x - 5} \) \( [-2, 5) \cup (5, \infty) \)

2. If \( f(x) = x^3 - 6x \), then \( f'(x) = 3x^2 - 6 \). Find each value of \( x \) at which the graph of \( f(x) \) has a horizontal tangent line.

   A. \( x = -6 \) only

   B. \( x = 0 \) only

   C. \( x = 0 \) and \( x = -6 \)

   D. \( x = \sqrt{2} \) only

   E. \( x = -\sqrt{2} \) and \( x = \sqrt{2} \)

3. Let \( f(x) = \begin{cases} \frac{x^2 - 5x + 6}{x - 3} & \text{if } x \neq 3 \\ \frac{4}{x - 3} & \text{if } x = 3 \end{cases} \). Evaluate \( \lim_{x \to 3} f(x) \).

   A. 1

   B. -3

   C. 4

   D. 2

   E. Does Not Exist
4. What is the domain of the function $f(x) = \sqrt{\frac{x}{1-x^2}}$?
   
   A. $(0, \infty)$  
   B. $[0, 1)$  
   C. $(-1, 1)$  
   D. $(-1, 0] \cup (1, \infty)$  
   E. $(-\infty, -1) \cup [0, 1)$  

   Questions 6 - 14 are worth 5 points each.

5. The supply and demand functions for a certain product are $p = x^2 + 9x + 40$  
   and $p = 70 - 2x^2$ respectively, where $p$ is the price of the product and $x$ is the number of  
   items measured in hundreds. Find the equilibrium quantity and price.
   
   A. 300 units; $52  
   B. 200 units; $62  
   C. 200 units; $38  
   D. 500 units; $20  
   E. 500 units; $110

6. The Intermediate Value Theorem guarantees that the equation $x^3 + 2x^2 - 2 = 0$ has a  
   solution on which of the following intervals?
   
   A. $(-2, 0)$  
   B. $(-2, -1)$  
   C. $(0, 1)$  
   D. $(-1, 0)$  
   E. $(1, 2)$

7. Find the value of $k$ so that the line passing through the points $(4, 2k + 9)$ and $(k, 2)$ is  
   perpendicular to the line $x + 3y - 6 = 0$.
   
   A. $-\frac{17}{7}$  
   B. $-\frac{2}{3}$  
   C. 1  
   D. $-5$  
   E. 19
8. If \( f(x) = |x| \), which function below best represents the given graph?

\[
\begin{array}{ccc}
\text{A. } y = 2 - f(x - 3) & \text{B. } y = 2 - f(x + 3) & \text{C. } y = f(x + 3) - 2 \\
\text{D. } y = f(x - 3) + 2 & \text{E. } y = 3 - f(x + 2) \\
\end{array}
\]

9. Let \( p = \lim_{x \to \infty} \frac{5 - 3x^4}{6 - 3x + x^3} \) and \( q = \lim_{x \to +\infty} \frac{1 - x^3}{(x^2 - 2)(2x + 1)} \). Find \( p \) and \( q \).

\[
\begin{array}{cc}
\text{A. } p = +\infty \text{ and } q = -1/2 & \text{B. } p = -\infty \text{ and } q = -2 \\
\text{C. } p = -3 \text{ and } q = -2 & \text{D. } p = 5/6 \text{ and } q = -2 \\
\text{E. } p = -\infty \text{ and } q = -1/2 \\
\end{array}
\]

10. Evaluate \( \lim_{x \to 2^+} \frac{10 - 5x}{|2 - x|} \)

\[
\begin{array}{cc}
\text{A. } 5/2 & \text{B. } -5/2 \\
\text{C. } -5 & \text{D. } 5 & \text{E. The limit does not exist.} \\
\end{array}
\]
11. Write the equation of the circle with center \((-4, 1)\) and containing the point \((0, -2)\).

A. \((x - 4)^2 + (y + 1)^2 = 25\)
B. \((x + 4)^2 + (y - 1)^2 = 25\)
C. \(x^2 + (y - 2)^2 = 5\)
D. \((x + 4)^2 + (y - 1)^2 = 5\)
E. \((x - 4)^2 + (y - 1)^2 = 5\)

12. Which of the following statements is/are true of the function below?

\[ f(x) = \begin{cases} 
5 & \text{if } x < 1 \\
1 + \frac{4}{x} & \text{if } 1 < x \leq 4 \\
\frac{\sqrt{x}}{} & \text{if } x > 4 
\end{cases} \]

I. \(f(x)\) has one nonremovable discontinuity.
II. \(f(x)\) is continuous at \(x = 4\).
III. \(f(x)\) has one removable discontinuity.

A. I only  
B. III only  
C. I & II only  
D. II & III only  
E. I & III only

13. If \(f(x) = \frac{\sqrt{2x+1} - \sqrt{3}}{x-1}\), find \(\lim_{x \to 1} f(x)\).

A. \(\frac{2}{\sqrt{3}}\)  
B. 2  
C. \(\frac{1}{2\sqrt{3}}\)  
D. \(\frac{1}{3}\)  
E. \(\frac{1}{\sqrt{3}}\)

4B
1. Let $f(x) = x^2 - 3x + 2$.

(a) Use the limit definition of derivative to find $f'(x)$.

(b) Write the equation of the tangent line to $y = f(x)$ at $x = 0$. Write you answer in slope-intercept form.
2. Let
\[ f(x) = \begin{cases} 
-1 & \text{if } x < -1 \\
\frac{x + 1}{3} & \text{if } -1 < x < 2 \\
|x| + 1 & \text{if } x > 2 
\end{cases} \]
(a) Evaluate the following limits; use symbols \( \infty, -\infty \) or D.N.E. when appropriate:

\[ \lim_{x \to -1} f(x) = \quad \lim_{x \to -1^+} f(x) = \quad \lim_{x \to 1} f(x) = \quad \]

\[ \lim_{x \to 2^-} f(x) = \quad \lim_{x \to 2^+} f(x) = \quad \lim_{x \to 2} f(x) = \quad \]

(b) Sketch the graph of \( f(x) \) on the axes below.

(c) List each value of \( x \) at which \( f(x) \) is discontinuous, and describe each as removable, or nonremovable. If the discontinuity is removable, state how you can define or redefine \( f(x) \) to make the function continuous there.
3. A manufacturer has a monthly fixed cost of $20,000 and a production cost of $4 for each unit produced. The product sells for $6 per unit. Let \( x \) denote the number of units.

(a) What is the cost function?

(b) What is the revenue function?

(c) What is the profit function?

(d) What is the \textbf{break-even} point?

(e) How many units must be sold to make a profit of 40,000 dollars?
4. While reviewing his accounts over the past year, the manager of a mall T-shirt stand made the following observations. During a normal week, he sold an average of 40 Gator T-shirts at a price $18 each. When he reduced the price by $3 for a clearance sale, an average of 10 more T-shirts sold per week.

(a) Find a linear model expressing demand $x$, the average number of T-shirts sold weekly as function of $p$, the price of a T-shirt.

\[ x(p) = \text{______________} \]

(b) One week the manager sold 54 T-shirts. Use your function to find the selling price of a T-shirt that week.

\[ $\text{__________} \]

(c) Find the corresponding revenue function where revenue is written as a function of price $p$.

\[ R(p) = \text{______________} \]